

# Response Surface Methods Applied to Submarine Concept Exploration

by

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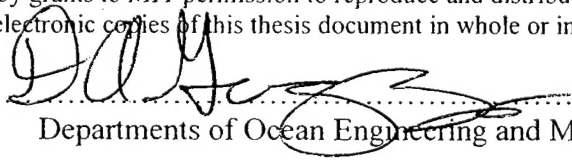
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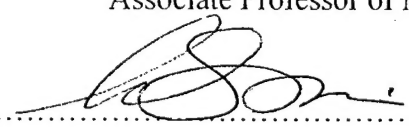
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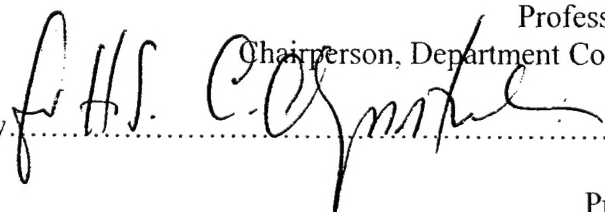
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## Abstract

It is estimated that 70 to 85 percent of a naval ship's life-cycle cost is determined during the concept exploration phase which places an importance in the methodology used by the designer to select the concept design. But trade-off studies are guided primarily by past experience, rules-of-thumb, and designer preference. This approach is ad hoc, not efficient and may not lead to an optimum concept design. Even worse, once the designer has a "good" concept design, he has no process or methodology to determine whether a better concept design is possible or not.

A methodology is required to search the design space for an optimal solution based on the specified preferences from the customer. But the difficulty is the design space, which is non-linear, discontinuous, and bounded by a variety of constraints, goals, and thresholds. Then the design process itself is difficult to optimize because of the coupling among decomposed engineering disciplines and sub-system interactions. These attributes prevent application of mature optimization techniques including Lagrange multipliers, steepest ascent methods, linear programming, non-linear programming, and dynamic programming.

To further improve submarine concept exploration, this thesis examines a statistical technique called Response Surface Methods (RSM). The purpose of RSM is to lead to an understanding of the relationship between the input (factors) and output (response) variables, often to further the optimization of the underlying process. The RSM approach allows the designers to find a local optimal and examine how the design factors affect the response in the region around the generated optimal point. RSM can be applied to submarine concept exploration and provide a methodology to: determine the optimal concept design based on customer preference, efficiently perform trade-off studies, determine the feasible design space, and the ability to determine in advance if a specified concept design is feasible and meets all the customer thresholds and constraints.

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# Chapter 1: Introduction

A submarine concept design is a difficult and iterative task due to the number of physical constraints, engineering design considerations, and customer requirements that must be satisfied. Because the number of mathematical equations required for a basic concept design is vast, any attempt to optimize the concept design process poses its own set of problems. The underlying equations to model a basic concept design are both linear and non-linear with continuous and discrete variables. The design process is difficult to optimize because of the coupling among decomposed engineering disciplines and sub-system interactions. These attributes prevent application of mature gradient-based optimization techniques including Lagrange multiplier, steepest ascent methods, linear programming, non-linear programming and dynamic programming. Genetic algorithms have been investigated [1], but have not proven to be a successful application tool.

## 1.1 Concept Design Studies

First, a designer will be challenged with a set of customer requirements and will attempt to produce a feasible and balanced submarine design. The process starts with estimates of the of the multiple requirements as the designer proceeds around the “design spiral” (see Figure 1) where each spoke represents one of the many disciplines and engineering requirements that needs to be considered. For instance, the spokes could represent the following: customer requirements, volume requirements, initial sizing, weight estimates, weight/buoyancy balancing, longitudinal balance, vertical balance, equilibrium polygon, propulsion, structures, maneuvering and cost. This philosophy is extensively discussed in the open literature, for example Brown [2]. For the first iteration, all calculations are entirely estimates. As the spiral of design progresses, refinements are made in each of the calculations as the design approaches a balanced condition. Once complete, the designer will have produced one balanced concept design.

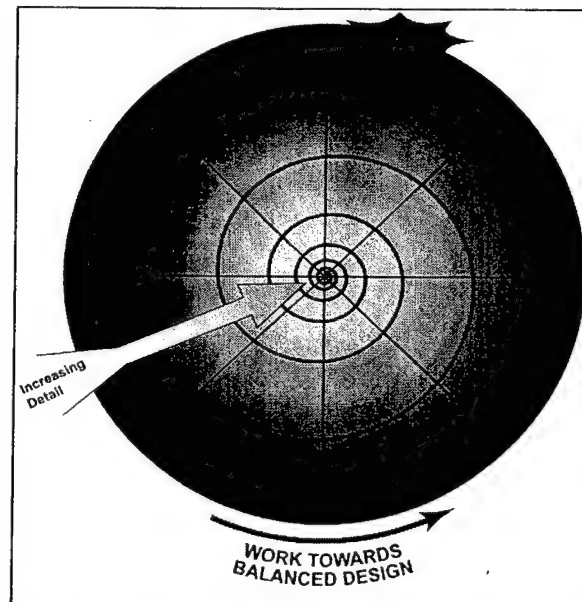


Figure 1: Design Spiral

Using the one balanced concept design, the designer can begin to perform trade-off studies. This is accomplished using one or both of two methods: the one-factor-at-a-time design approach or the Pareto plot.

### **1.1.1 One-Factor-At-A-Time Design Approach**

To evaluate the current concept design, the designer will compare his<sup>1</sup> design to the customer requirements. But if the design does not produce the required results, the designer will make changes to the baseline concept design. By selectively changing the design parameters or factors, the designer is using his experience to select the correct factor to improve the performance of the concept design. Each time the designer makes changes to the design, he must then proceed around the design spiral to attain a balanced concept design, if possible. If a feasible solution is achievable, there is no guarantee the new concept design will be an improvement over the last concept design.

With this one-factor-at-a-time design approach, there is no systematic or specific methodology being applied. The designer is developing point designs in an educated manner, but he is not using a specific methodology. Instead, the designer is relying on his past experience and understanding of submarine concepts to identify the correct design factors to change and making the appropriate changes. Yet this approach is ad hoc, and the resulting process is highly inefficient, not systematic, and may not lead to an

---

<sup>1</sup> In the interest of brevity, the personal pronoun will be "he," even though "he or she" is understood by the author and reader.

optimum (defined as meeting the customer requirements at the least cost) design solution. Furthermore, once the designer has a “good” concept design, he has no process or methodology to determine whether a better concept design is possible or not.

### 1.1.2 Pareto Plot

This method uses the Overall Measure of Effectiveness (OMOE) as a means to determine the optimal solution. An excellent reference on the Pareto plot is by Whitcomb [3]. To generate the Pareto plot, the designer develops numerous concept designs and determines the OMOE and cost for each. By plotting each point design on the Pareto plot (see Figure 2), the designer can examine his various designs and have a graphical comparison. If a cost constraint is imposed, the designer can determine from this graph the best concept design (the highest OMOE under the cost constraint).

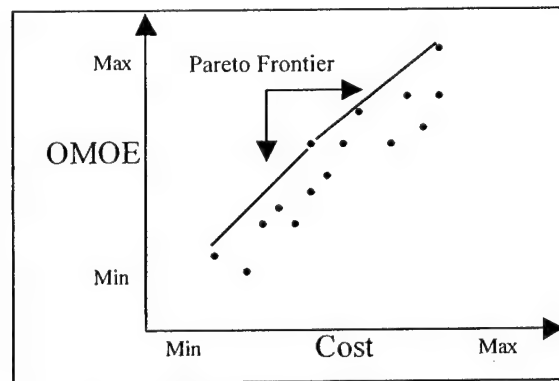


Figure 2: Pareto Plot

The Pareto plot can be a very useful tool, but it also has its limitations. One limitation is the determination of the Pareto frontier. In theory, the designer would produce an infinite number of concept designs and plot each on the Pareto plot to

determine the Pareto frontier. But this task is close to impossible. The designer would only produce enough point designs until some form of the Pareto frontier is visible. But the difficulty lies in the fact that the designer cannot say with 100 percent certainty that he has determined the frontier. At a specific cost, the designer will have a design with the highest OMOE, but he is not certain if there is another design at the same cost with an even higher OMOE.

Another limitation of the Pareto plot is the inability to examine other point designs. To examine another point design, the designer will have to selectively change the design factors and rebalance the design by using the design spiral. Or if the designer would like to examine a point design of a specific cost and OMOE, the designer would have to estimate the correct design parameters to produce a concept design at the required cost and OMOE. The first attempt would miss its mark, so the designer will have to iterate until he attains the correct cost and OMOE.

The one-factor-at-a-time design approach and the Pareto plot are limited. These limitations are further amplified when considering the concept exploration phase of the acquisition cycle.

## ***1.2 Concept Exploration Phase of the Acquisition Cycle***

The U. S. Navy estimates that 70 to 85 percent of the life cycle cost of major acquisition programs is determined upon the completion of the concept exploration phase [4]. It is therefore critical that the selected concept design not only satisfy the physical constraints, engineering design considerations and customer requirements, but at the lowest life cycle cost. But how is this achieved? The designer can use the one-factor-at-

a-time approach or the Pareto plot discussed in the previous section, but the designer cannot guarantee that the selected design is optimal, based on customer requirements, at the lowest cost.

Even if the designer can produce an optimal concept design, the designer has no method to examine the impact of changing one design factor and its impact on the balanced design. Neither of the two methods previously discussed provide a means of easily evaluating changes in the design factors. For instance, if the designer decides to add an additional 40 feet to the parallel mid-body to accommodate a larger modular payload section, he has no other method available other than to produce another point design via the design spiral. Or if it is determined that the current design is one knot below the Top Level Requirement (TLR) for speed, the designer has no methodology for a course of action other than make point changes and re-balancing the design.

In summary, a methodology is needed to allow the designer to systematically optimize the concept design, efficiently perform trade-off studies, and examine the impact on the design when a design factor is changed. A statistical tool called Response Surface Methods can accomplish (and even surpass) this challenge.

### ***1.3 Response Surface Methods***

Response Surface Methods (RSM) has been successfully used since the 1950s on a wide variety of problems in chemical engineering, agriculture, chemistry and mechanical engineering [5][6]. But with faster computer processor speed, computer simulation of physical processes has become a standard tool of many design and manufacturing engineers. Powerful computer-aided design tools, finite element analysis

programs, and high-level deterministic and stochastic simulation packages make possible computations and detailed analysis of engineering programs not dreamed of years ago.

With the development of large-scale mathematical models of engineering designs, RSM can now be applied to the model to optimize the design [7]. For instance, the Aerospace Systems Design Laboratory (ASDL) at Georgia Institute of Technology is using RSM to optimize the complex design of aircraft.

Using RSM methodology, the designer will produce an  $n$ -dimensional surface using a group of techniques in the empirical study of relationships between one or more measured responses (the output variables) and a number of factors (the input variables). This surface represents all feasible and balanced designs. To create this surface, the designer will be required to produce a finite number of point designs. The selected point designs are very specific and are determined by the type of response surface design model selected not the designer. Most importantly, once the response surface is created, the designer can examine any other point design by moving along the surface. The designer is not required to start at the beginning using the design spiral. Along this surface the designer can find the optimal solution and effectively perform trade-off studies.

In general, RSM is used to answer the following questions:

1. How do the design factors affect the specific response?
2. What values, if any, of the factors will produce a response simultaneously satisfying specified constraints, thresholds, and goals?
3. What values of the factors will produce an optimal design, and what is the response surface like close to this optimal solution?

## ***1.4 Response Surface Methods Applied to Submarine Design***

This chapter illustrates the need for an improvement in current submarine concept design practices. With the combination of a submarine mathematical model and a powerful statistical software package, RSM can be applied to submarine concept design and can yield great insight. Once the response surface is created, the designer can move along the surface to examine the impact of changing design factors on the response, perform trade-off studies, and determine the optimal solution (based on customer requirements). For instance, if RSM is applied to the Pareto plot, a response surface can be generated for both the cost and OMOE. Once the response surface is created, the designer can move along the surface and easily determine the optimal OMOE for a specified cost.

The application of RSM to submarine concept exploration is the purpose of this thesis. This study is structured in the following format: Chapter Two will provide a basic understanding of RSM and application of this technique. Readers will be provided two references if they desire further information. To generate the concept designs needed to create the response surface, a mathematical model must be used. Chapter Three will discuss the mathematical model used to generate the data for this thesis. Chapter Four will show the results from applying RSM to a submarine concept design, and will also discuss lessons learned from this research and the limitations of applying RSM. Chapter Five will then summarize the application of RSM to submarine concept design and provide direction for further work.



## Chapter 2: Response Surface Methods

This chapter will review the basic fundamentals of RSM. If the reader needs a more detailed explanation, there are two excellent reference books. “Understanding Industrial Designed Experiments,” by Schmidt and Launsby [8] focuses on the application of RSM to current design problems, but only reviews the basics of the statistical techniques used in RSM. For a more detailed explanation of the statistics, “Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building,” by Box and Hunter [9] provides an excellent statistical foundation.

### ***2.1 Terminology***

The following terminology is used in RSM:

- **Factors:** The input variables or design parameters. Represented by capital letters (A, B, C) or  $x_i$ .
- **Levels:** The different settings for each factor. For a two-level factor, the low level is represented by (-1) and the high level as (+1). For a three level factor, the intermediate level is represented by (0).
- **Response:** The output of interest, represented by the letter y.

- Interaction(s): Refer to dependencies between a factor's effect on the response and levels of another factor. The interaction of A and B is represented as AB.

## 2.2 Overview of RSM

As stated in Chapter One, RSM will produce an n-dimensional surface using a group of techniques in the empirical study of relationships between one or more measured responses and a number of factors. Two such methods, the Box-Behnken and the Central Composite design, perform a quadratic fit between k design factors and the response using the following second-degree polynomial approximation:

$$y = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i=1}^k b_{ii} x_i^2 + \sum_{i=1}^k \sum_{j=i+1}^k b_{ij} x_i x_j + \varepsilon. \quad (1)$$

The coefficients,  $b_0$ ,  $b_i$ ,  $b_{ii}$ , and  $b_{ij}$ , are easily obtained from a multivariate regression software package. The error term  $\varepsilon$  represents pure error and lack of fit. If the quadratic surface does not accurately fit the data, as indicated by the lack of fit error term, either the design space is too large or the incorrect factors were selected. If the designer determines the poor fit is caused by a large design space, he can improve the fit by reducing the design space; this is achieved by reducing the range for each factor. For instance, if initially factor A had the low level (-1) set at 100 and the high level (+1) set at 200, then setting (-1) at 30 and (+1) at 70 will improve the accuracy of the quadratic fit. Once a "good" quadratic fit is attained, this quadratic surface represents all feasible concept designs.

To apply RSM, one would proceed through the following three steps:

1. Engineering Model: Build a mathematical model of the design, and identify the potential factors for each response of interest.
2. Screening Experiment: Determine which factors are critical, i.e. those factors that have a statistical impact on the response.
3. Response Surface Modeling: Within the design space, create a quadratic surface for the response as a function of the critical factors.

### **2.3 Engineering Model**

The engineering model is a critical aspect of RSM. The engineering model must contain enough fidelity to provide useful results. Once the level of detail is in the model, the designer must then select the response(s) he is interested in investigating and optimizing. There can be one or more responses. When there are multiple responses, the steps outlined in this chapter must be applied to each response. Chapter Four will examine five responses related to submarine concept design.

Once the responses are identified, the designer will need to determine all possible sources of variation for each response. The sources of variation can be grouped into three categories:

1. Controllable: the designer can control this source of variation. For example, if the response of interest is submerged displacement for a submarine design, then the designer should ensure that all the designs are weight limiting or volume limiting. By controlling this source of variation, the response accuracy will improve.
2. Noise: the designer cannot control this source of variation.
3. Factors: the sources of variation that the designer wants to investigate.

If other sources of variation exist (unknown to the designer), the accuracy of predicting other point designs by moving along the surface will be degraded. The loss of accuracy is dependent on the magnitude of the unaccounted variation source, and will be indicated by the quadratic curve fit statistical results.

Many industrial experiments have failed because all sources of variation were not identified prior to the start of conducting RSM. A good design will identify the factors and all controllable and noise variations ahead of time. Using the statistical techniques of randomization, repetition, and blocking, the designer has the ability to reduce or eliminate unwanted sources of variation. Yet this must be arranged before applying RSM. If the designer does not fully understand all the statistical tools available and does not use them to conduct well-designed experiments, then he may be misled by the results.

## ***2.4 Screening Experiment***

The purpose of the screening experiment is to determine which factors identified in Section 2.3 are critical (factors that statistically effect the response). The most efficient way to identify these factors is to use an experimental design process called Design of Experiments (DOE). By applying the methodology of DOE, the designer systematically develops numerous design variants to observe the corresponding changes in the response. By collecting the data and analyzing the results, the designer uses statistical analysis to estimate the effect of each factor and their interactions on the response. Using the statistical tools ensures accuracy and validity. Overall, the DOE

methodology is a scientific approach applied to a concept design by the designer to determine how the factors effect the response.

For the screening experiment, only two levels per factor are used to estimate the effect of each factor on the response (three or five levels per factor are used when developing the quadratic surface for RSM). It is critical that the levels for each factor are within the design space of interest. The importance of this will be demonstrated in Chapter Four.

To determine the critical factors, DOE consists of the following steps to be discussed in more detail:

1. Experiment Setup: Depending on the number of factors and required resolution (to be discussed), the number of variants  $n$ , or experiment runs, to be developed is determined. More important, the factor levels for each variant are predetermined using orthogonal arrays.
2. Execution: The designer produces  $n$  variants.
3. Analysis: Since orthogonal arrays were used to develop the variants, statistical analysis can be used to estimate the effect of each factor and their interactions on the response.

### **2.4.1 Experiment Setup**

One option to determine the required number of experiment runs, or design variants, is the full factorial design matrix. The full factorial design matrix requires the designer to produce a variant for each possible combination of the factors. The number of variants,  $n$ , for a two-level screening experiment as a function of  $k$  factors is

$$n = 2^k. \quad (2)$$

Table 1 is an example of a full factorial design matrix ( $k = 3$ ) and includes every possible combination of the factors. As shown in Table 1, (-1) is the coded value for the low level and (+1) is the coded value for the high level. The reasoning for the selection of (-1) and (+1) for the low and high levels is to verify that the design matrix is orthogonal and to generate the factor interaction columns (both to be explained shortly). The factors A, B, and C could, for example, correspond to temperature, pressure, and time.

Run	A	B	C
1	-1	-1	-1
2	-1	-1	+1
3	-1	+1	-1
4	-1	+1	+1
5	+1	-1	-1
6	+1	-1	+1
7	+1	+1	-1
8	+1	+1	+1

**Table 1: Full Factorial ( $k=3$ ) Design Matrix**

The advantage of the full factorial design matrix is the ability to estimate the effect of all factor and factor interactions on the response. To further expand upon the example in Table 1, the three-factor full factorial design matrix can estimate the effects of the following:

- Three linear or main effects (A, B, C),
- Three two-way interactions (AB, AC, BC),

- One three-way interaction (ABC).

Depending on the engineering model, the factor interactions' (two-way, three-way, etc.) effects may be statistically significant, and their impact on the response can be determined with the full factorial design matrix.

As mentioned earlier, full factorial design matrices use orthogonal arrays to estimate the effects of each factor and their interactions independently of others. Mathematically, the requirements of an orthogonal array are:

1. Sum of each column is zero.
2. The dot product of any two columns is zero.

Another way to visualize orthogonal arrays is shown with Table 1. When A is at (-1), the remaining columns have a balanced number of (+1) and (-1) values. This also applies for all other columns evaluated at their (+1) and (-1) levels. This balancing property results in an orthogonal design, which allows the designer to estimate the effects of each factor and factor interactions independently of the others.

With the orthogonal design matrix, the statistical estimation of the factor effect on the response is simple to compute (for a more detailed explanation, see reference [7]). For the specific column associated with the factor of interest, the designer should determine the average of the responses associated with the (-1) values and average for the (+1) values. The difference in the average for the (-1) and the (+1) is a measure of the

effect of that factor on the response. The designer can use either the t-test or the F-test to determine if the difference is statistically significant.

Table 1 has a column for each factor, which allows the designer to examine the effect of each factor on the response using the method described in the previous paragraph. To estimate the effect of the factor interactions, one must produce similar columns for each factor interaction. With the (+1) and (-1) notation, the factor interaction AB column is simply the product of columns A and B, row by row. Table 2 shows all the possible factor interactions of the example in Table 1. By visual inspection of Table 2, the factor interaction columns are orthogonal and allow the designer to estimate the effect of the factor interactions on the response.

Run	A	B	C	AB	AC	BC	ABC
1	-1	-1	-1	+1	+1	+1	-1
2	-1	-1	+1	+1	-1	-1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	-1	+1	+1	-1	-1	+1	-1
5	+1	-1	-1	-1	-1	+1	+1
6	+1	-1	+1	-1	+1	-1	-1
7	+1	+1	-1	+1	-1	-1	-1
8	+1	+1	+1	+1	+1	+1	+1

**Table 2: Full Factorial Design Matrix with Factor Interactions**

The most serious disadvantage of the full factorial design is that the number of variants can become quite large as the number of factors increases. In most experiments, interactions beyond two ways are not significant. Therefore it is possible to use a fractional factorial design matrix to reduce the number of variants required to estimate



the effect of the factors on the response. But since one would be performing a subset of the full factorial design, only a portion of the factor interaction effects can be estimated.

The fractional factorial design matrix allows the designer to estimate all linear effects and desired factor interactions while requiring fewer runs than the full factorial. The number of runs for a fractional factorial is

$$n = 2^{k-q}, \quad (3)$$

where  $q = 1$  indicates a half fraction,  $q = 2$  a quarter fraction, etc. The advantage of the fractional factorial is the number of runs required to estimate all linear effects and specified interactions is smaller. Like the full factorial, the fractional factorial design matrices are orthogonal and therefore allow accurate estimation of the factor and specified factor interaction effects on the response.

Since the fractional factorial does not require all possible factor combinations, some of the higher order factor interaction effects become non-estimable. An effect is non-estimable when it is confounded, or aliased, with another effect. For example, consider a  $2^{4-1}$  fractional factorial. To perform a full factorial would require 16 experiment runs, but by performing a half fraction, only eight experiment runs are required. To generate the design matrix for the fractional factorial, one should first generate the design matrix for a  $2^3$  showing columns for each factor and factor interactions (Table 2). To generate the coded settings for the fourth factor D, one could use the factor interaction column ABC as shown in Table 3. Therefore, when the experiment is conducted, the factor interaction column ABC is used to determine the

setting for factor D. By doing so, one has confounded factor D with the factor interaction ABC (represented by  $D=ABC$ ).

Run	A	B	C	AB	AC	BC	D=ABC
1	-1	-1	-1	+1	+1	+1	-1
2	-1	-1	+1	+1	-1	-1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	-1	+1	+1	-1	-1	+1	-1
5	+1	-1	-1	-1	-1	+1	+1
6	+1	-1	+1	-1	+1	-1	-1
7	+1	+1	-1	+1	-1	-1	-1
8	+1	+1	+1	+1	+1	+1	+1

**Table 3:  $2^{4-1}$  Fractional Factorial Design Matrix**

In the example of Table 3, the main factor D is confounded with the factor interaction ABC, or  $D = ABC$ . (Using the equal sign does not imply that the D and ABC effects are the same, but that the same column is used to represent D and ABC.) By assigning  $D = ABC$ , the evaluation of the D and ABC column effects cannot be separated. When the experiment is complete and if the column effect is determined to be statistically significant, it is either caused by factor D, factor interaction ABC, or some combination of the two. If the designer assumes the ABC interaction is unlikely to be important, then the column measures the effect of factor D. If the designer believes this assumption is not true, the designer should select another design matrix.

By using the factor interaction column ABC to generate the factor D levels in Table 3, the designer has created additional confounded interactions other than  $ABC = D$ . The determination of the remaining confounded factor interactions is easily determined.

If any column is multiplied by itself, the resulting column contains all (+1) values, called the identity column, I. Using this relation, the “defining relation” for the example in Table 3 is attained as follows:

1. Start with  $D = ABC$
2. Multiply each side by D and use  $DD = D^2 = I$
3.  $DD = I = ABCD$
4. The defining relation is  $I = ABCD$

With the defining relation defined for the example in Table 3, other confounded effects are determined by multiplying each factor and their interactions by the defining relation. Using the relation  $AA = BB = CC = DD = I$ , the simplified equation is the confounded relationship. Table 4 shows the alias pattern for the example in Table 3.

Effect	Math	Alias
A	$AI = AABCD = BCD$	$A = BCD$
B	$BI = BABCD = ABBCD = ACD$	$B = ACD$
C	$CI = CABCD = ABCCD = ABD$	$C = ABD$
D	$DI = DABCD = ABCDD = ABC$	$D = ABC$
AB	$ABI = ABABCD = AABBCD = CD$	$AB = CD$
AC	$ACI = ACABCD = AABCCD = BD$	$AC = BD$
AD	$ADI = ADABCD = AABCDD = BC$	$AD = BC$
BC	$BCI = BCABCD = ABBCCD = AD$	$BC = AD$
BD	$BDI = BDABCD = ABBCDD = AC$	$BD = AC$
CD	$CDI = CDABCD = ABCCDD = AB$	$CD = AB$
ABC	$ABCI = ABCABCD = AABBCDD = D$	$ABC = D$
ABD	$ABDI = ABDABCD = AABBCDD = C$	$ABD = C$
ACD	$ACDI = ACDABCD = AABCCDD = B$	$ACD = B$
BCD	$BCDI = BCDABCD = ABBCCDD = A$	$BCD = A$
ABCD	$ABCDI = ABCDABCD = AABBCDD = I$	$ABCD = I$

Table 4: Alias Pattern For The Example In Table 3

Upon examination of Table 4, all factor effects are confounded with three-way interactions, and all two-way interactions are confounded with other two-way

interactions. Since in most designs three-way interactions are not statistically significant, the alias of the factor effects with the three-way interactions should not pose a problem. Yet on the other hand, the alias of the two-way interactions with other two-way interactions needs to be evaluated for the specific design being investigated. The key to using fractional factorial design matrices is to purposely plan which interaction effects are confounded.

To describe the alias of factor and factor interactions, the term resolution is used for fractional factorial designs. The definitions are as follows:

- Resolution III:
  - Main effects are not confounded with one another.
  - Main effects confounded with two-way interactions.
- Resolution IV:
  - Main effects are not confounded with one another.
  - Main effects are not confounded with two-way interactions.
  - Main effects confounded with three-way interactions.
  - Two-way interactions confounded with other two-way interactions.
- Resolution V:
  - Main effects are not confounded with one another.
  - Main effects are not confounded with two-way interactions.
  - Main effects are not confounded with three-way interactions.
  - Two-way interactions are not confounded with one another.

Using this terminology, the example in Table 3 is a fractional factorial design with resolution IV.

In summary, the experimental setup of the screening experiment is an important step. The goal is to determine the critical factors, and this can be accomplished by using a full factorial or a fractional factorial. If the fractional factorial design matrix is selected,

the designer must carefully select which interactions are confounded, and this is determined by the engineering design under consideration.

### **2.4.2 Execution**

Now that the experiment setup is complete, the designer can execute the plan. The designer will develop each variant with the factor levels specified by the design matrix selected. As the designer goes around the design spiral for each variant, he should be checking the design characteristics for any unusual results or unaccounted variance. For each balanced design, the selected response is recorded for the analysis phase.

### **2.4.3 Analysis**

With the variants complete, the designer now examines the results using a statistical package. For this research, the statistical package JMP<sup>®</sup> by SAS institute is selected to analyze the data. Guided by the results and graphical displays of the statistical package, the designer selects those factors and factor interactions that are statistically significant (to be illustrated in Chapter Four). These critical factors will then be selected to develop the response surface.

## ***2.5 Response Surface Modeling***

The designer now develops a curved surface (quadratic) as a function of the critical factors. To create this surface, the designer has two options: the Central Composite and Box-Behnken design. This surface represents all feasible designs within the design space defined by the range of the critical factors. Once the surface is created,

the designer can easily examine other point designs by moving along the surface and does not need to revert back to the design spiral.

The Box-Behnken design is a three-level, nearly orthogonal, resolution V design used for modeling factors with three-levels. The slight non-orthogonal design matrix is not a concern if the analysis is conducted using least squares regression. As shown in Figure 3 for three factors, the Box-Behnken design does not include any corner points of the design space. Therefore if the corner points are infeasible, the Box-Behnken design should be used to generate the quadratic curved surface. Yet the disadvantage is that the Box-Behnken design will produce a higher uncertainty of prediction near the corner points.

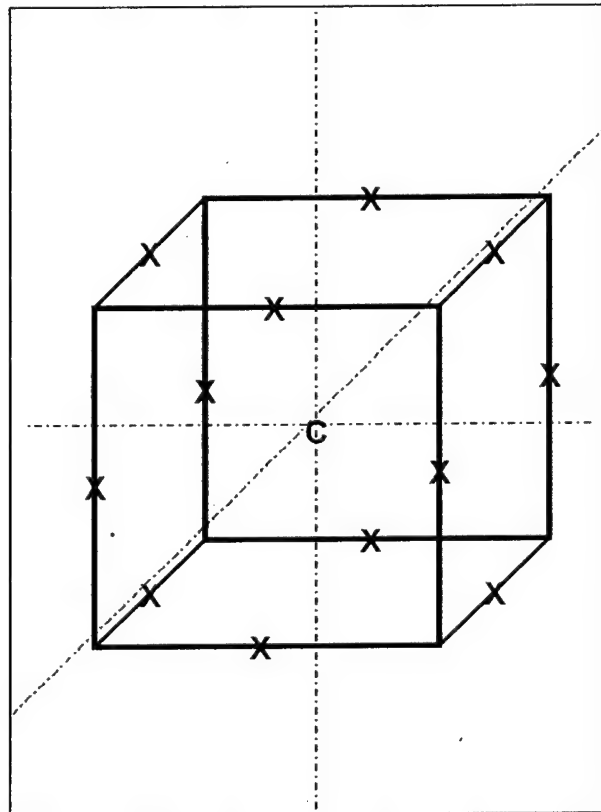


Figure 3: Three Factor Box-Behnken Design

The Central Composite design is a three- or five-level design used to create the quadratic curved surface by including the center, corner, and axial points of the design space (see figure 3 for a three-factor Central Composite design). The axial points are at a specified distance from the center, which means that  $(+1)$  or  $(-1)$  no longer represent the factor maximum or minimum. Since the Central Composite design uses the corner points of the design space, the generated curved surface will be more accurate (as compared to the Box-Behnken design) at these points, but this might strain the engineering model.

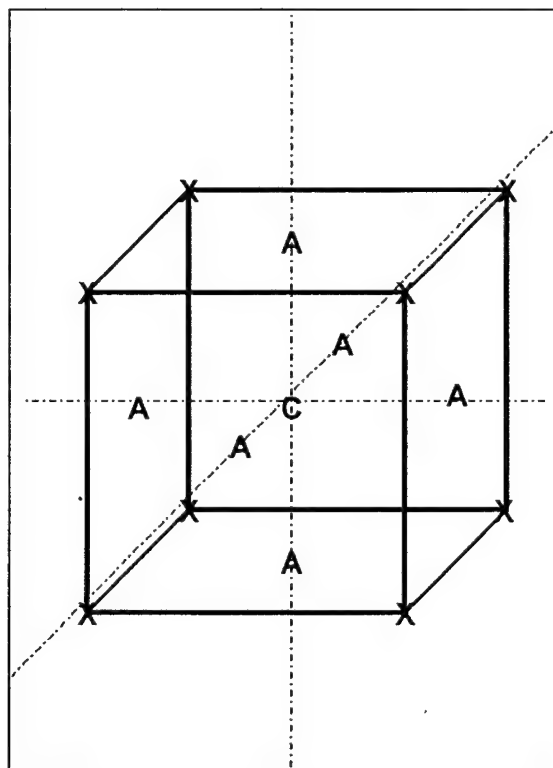


Figure 4: Three Factor Central Composite Design

Once the designer has created the quadratic surface using the Box-Behnken or Central Composite design, the statistical packages will provide the statistical results and graphical displays to determine the accuracy of the surface fit. This will be illustrated in Chapter Four.

## ***2.6 Response Surface Methods Summary***

This chapter has briefly explained RSM methodology. By applying the DOE screening experiment, the designer has a tool to systematically determine the critical factors within the design space of interest. With the critical factors identified, the designer can focus his attention on the critical factors and not on the other factors that have no impact on the response. Using the critical factors, the designer can create the response surface and efficiently perform trade-off studies. Using a statistical software package, the designer can use the response surface to examine an infinite number of design variants.



## **Chapter 3: Submarine Model**

The first step to apply RSM to submarine concept designs is to develop a mathematical model. The MIT Ocean Engineering 13A curriculum has been using a MathCAD Submarine Design Model for developing concept designs. Versions of the MathCAD Submarine Design Model have been used for numerous conversion and yearlong design projects as part of the curriculum. One such design project, “The Next Generation Nuclear Submarine (NGSSN): A Study in Modularity,” by Hanson and Hunt [10], modified the MathCAD Submarine Design Model for a modular submarine. The modular submarine model by Hanson and Hunt is used for this thesis to examine the application of RSM.

### ***3.1 Modular Submarine Model***

The main design objective of the modular submarine model, as defined in the NGSSN mission need statement, was to enable a rapidly re-configurable submarine platform which could incorporate new technologies and developments as they evolved. To achieve these objectives, a module payload section (64 ft in length) consisting of three

20 ft by 20 ft module bays was inserted in the center section of a modern submarine hull form, as shown in figure 5. The specifications of the resulting submarine are given in Table 5.

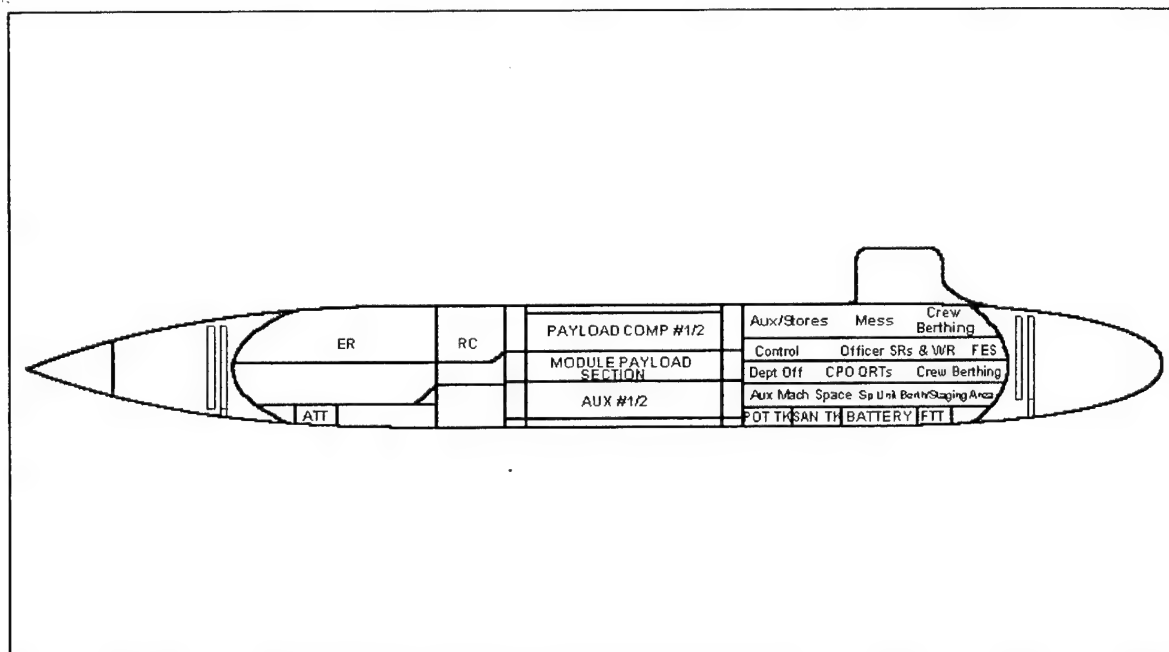


Figure 5: Baseline Ship Profile View

Parameter	Baseline Design
Displacement (surfaced)	8499 ltons
Displacement (submerged)	9562 ltons
Length	372.4 feet
Diameter	40 feet
SSTG's (combined)	7200 kW
Payload Section Length	64 feet
Installed Shaft Horse Power	28,100 shp
Speed (submerged)	28.08 knots
Endurance Range	90 days
Compliment	100

Table 5: Baseline Design Summary

Hanson and Hunt used an Overall Measure of Effectiveness (OMOE) as a framework for the evaluation of individual submarine designs in the preliminary trade-off design stage. The selected OMOE was a function of three design parameters: test depth, submerged speed, and payload length. By including the length of the modular payload length as one of the OMOE parameters, Hanson and Hunt addressed the concerns of platform mission modularity. Chapter Four selects this OMOE as one of the responses to be investigated.

If the reader needs more information on the project by Hanson and Hunt, see reference [10]. Appendix B of this reference includes the modified MathCAD Submarine Design Model used for the modular submarine. The same MathCAD model is used for this thesis to generate design variants.

### ***3.2 MathCAD Submarine Design Model***

The MathCAD Submarine Design Model uses the software program MathCAD by MathSoft. Using this software package, the designer directly inputs the mathematical equations, similar to Figure 6, into the document. This ease of use and the ability to quickly change the mathematical equations are advantages of using MathCAD. If the designer wants to evaluate a radically different design concept, the designer can easily modify the MathCAD Submarine Design Model.

$$\text{LCG}_{\text{LEADs}} := \frac{W_{\text{pb}} \cdot \text{LCG}_{\text{LEAD}} - W_{\text{pbm}} \cdot \text{LCG}_{\text{pb}}}{W_{\text{pbs}}}$$

**Figure 6: MathCAD Example**

The most significant disadvantage of the MathCAD Submarine Design Model is the manual iterations required by the designer to obtain a balanced design. For example, one section of the model determines the volume requirements based on specified design parameters. Then in another section, the designer “spins a hull” and determines the volume available with the specified hull. If the volume requirements and the hull volume do not match, the designer must iterate specified design parameters until a balance is achieved. Taking into account the other balancing requirements in the MathCAD Submarine Design Model, the designer will expend about four hours per variant, on average, manually manipulating the design parameters to attain a balanced submarine concept design.

### ***3.3 Submersible Design Program***

The author used the equations from the MathCAD Submarine Design Model to develop a computer program called the “Submersible Design Program.” Since the underlying equations are identical in the Submersible Design Program and the MathCAD Submarine Design Model, both design tools will generate identical concept designs. The difference, though, is the synthesis algorithm built into the Submersible Design Program. This synthesis algorithm performs all the manual iterations performed by the designer in the MathCAD Submarine Design Model. Using the Submersible Design Program, the designer inputs the design parameters and presses the “synthesis” button to activate the algorithm. With the guidance of the synthesis algorithm, the designer can produce a

balanced design in about five minutes compared to the four hours using the MathCAD Submarine Design Model.

The Submersible Design Program is programmed in JAVA, which is an object-oriented language with a vast library for graphical user interfaces (GUI). The advantages of the program include the following to be discussed in more detail: graphical user interface, object-oriented modularity, and the synthesis algorithm.

### **3.3.1 User Interface**

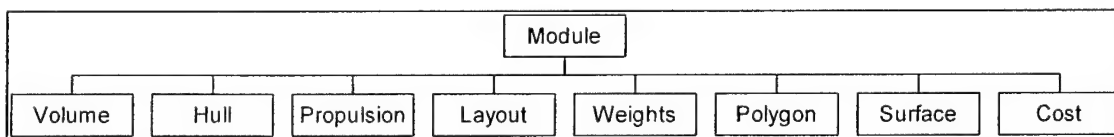
The JAVA language has a vast library for graphical user interfaces. Taking advantage of GUI library, the Submersible Design Program has a user-friendly interface. Via the interface, the designer can easily manipulate the design parameters for the concept design. With the data entered, the designer presses the “synthesis” button to attain a balanced design. Once a balanced concept design is attained, the program provides two graphs to aide the designer in the evaluation of the design:

1. Profile View: A profile view of the concept design, showing the location of compartments, bulkheads, decks, and variable ballast tanks.
2. Equilibrium Polygon: Plots the equilibrium polygon with the specified loading conditions.

### **3.3.2 Object-Oriented Modularity**

All calculations in the Submersible Design Program are grouped into classes by functionality as illustrated in Figure 7. The general description of each class is as follows:

- Volume: Calculates the volume requirements (operations compartment, engine room, pressure hull, main ballast tanks, submerged, etc.) based on design parameters.
- Hull: Generates a submarine hull.
- Propulsion: Calculates the submerged and surface speed based on the installed propulsion system and generated submarine hull.
- Layout: Specifies location of bulkheads, decking, and variable ballast tanks.
- Weights: Calculates the surfaced and submerged displacement. Locates lead to attain an even submerged trim.
- Polygon: Calculates the loading conditions for the equilibrium polygon and generates the equilibrium polygon plot.
- Surface: Calculates the surfaced draft, GM, and trim.
- Cost: Calculates the lead ship acquisition cost using a weight-based model.



**Figure 7: Submersible Design Program Abstract Class Architecture**

Using the architecture shown in Figure 7, the Submersible Design Program takes advantage of the JAVA object-oriented language to produce a modular program. Figure 8 illustrates the modular architecture of the program. The abstract superclass is Module, which defines the methods needed to interact with the user interface. By defining Module as an “abstract” class, any class that inherits Module must implement the methods defined in Module to interact with user interface. Therefore, when Hull class

inherits Module (indicated by the connecting line), the Hull class will have the required methods to interact with the graphical user interface. Similarly, any other class that inherits Module also has the ability to interact with the user via the graphical user interface.

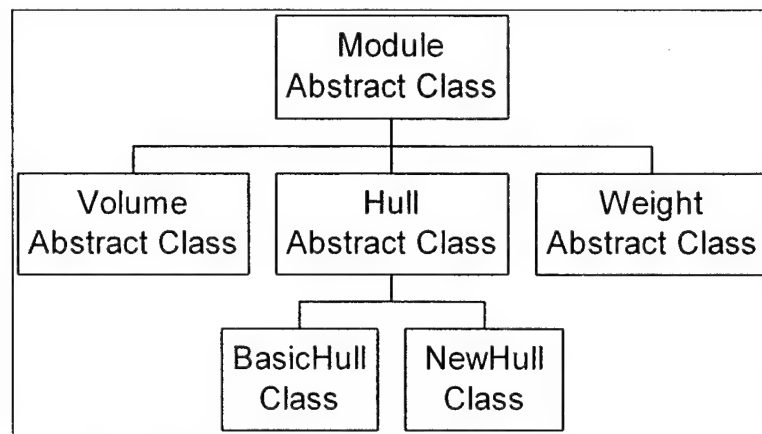


Figure 8: Submersible Design Program Modularity

The Hull class is used to generate the submarine hull including any appendages and the sail. Yet the key to the modularity is that the Hull class does not provide any of the mathematical modeling. By defining Hull as an abstract class, this class defines the required methods needed to interact with the synthesis algorithm. Therefore, any module that inherits Hull must implement the defined methods needed to interact with the synthesis algorithm. In Figure 8, BasicHull class contains the mathematical modeling to generate a submarine hull, and since it inherits the abstract class Hull, BasicHull can interact with the synthesis algorithm. And, just as important, since Hull inherits Module, so does BasicHull. In summary, BasicHull performs the mathematical modeling needed to generate a submarine hull and by using inheritance, BasicHull has the required

functions needed to interact with the synthesis algorithm and the user interface. If the reader desires more information on the object-oriented JAVA language, see reference [11].

With the architecture described in the preceding paragraph, the modularity works as follows: If the designer would like to examine a different hull form, he needs to simply develop the mathematical model to represent the new hull form. With the equations defined, the designer translates the equations into the JAVA language and calls the new class NewHull. Through inheritance, as shown in Figure 8, the NewHull class can be inserted into the Submersible Design Program and interact with the synthesis algorithm and the user interface.

The modularity of the Submersible Design Program allows the designer to add more detailed calculations or different attributes to the program and therefore analyze for different concept designs. In order to do this, the designer must know the JAVA programming language. On the other hand, if the design under consideration is radically different, the modularity might not work. In this case, the architecture of the program itself may need minor changes. If the work to modify the source code is significant, the designer may want to use the MathCAD model because it is very easy to modify.

### **3.3.3 Synthesis Algorithm**

The synthesis algorithm in the Submersible Design Program aides the designer in quickly attaining a balanced concept design. The algorithm executes each of the classes in the program and depending on the out-of-balance conditions, will change the design parameters to produce a balanced concept design. If the algorithm cannot produce a



balance design, the algorithm will terminate and inform the designer why. For example, if the designer selects a submarine diameter of 40 ft, but attempts to install five decks with a deck height of 12 ft, the synthesis algorithm will terminate and inform the designer. He can then adjust the number of decks, deck height, or diameter and rerun the synthesis algorithm.

With no errors, the synthesis algorithm takes about one minute to execute. If errors do develop, the designer, with the aide of the synthesis algorithm, can modify the design parameters until a balanced design is achieved. From start to finish, the designer can produce one balanced concept design in about five minutes with the aide of the synthesis algorithm.

### ***3.4 Data for Thesis***

The modular submarine model by Hanson and Hunt is used for the application of RSM. As stated earlier, the MathCAD Submarine Design Model and the Submersible Design Program will produce the same concept design. With the ability to quickly develop concept designs, this thesis used the Submersible Design Program for the generation of the design variants. To validate the Submersible Design Program, the MathCAD Submarine Design Model was used to verify the results.



## Chapter 4: Analysis

Using the modular submarine model from Chapter Three, this thesis examines the following responses: cost, submerged displacement, length, submerged speed and OMOE. The first four responses were selected since they are the main characteristics of a submarine design. The fifth response, OMOE, is used to examine the application of RSM to the Pareto plot (OMOE versus cost) and allow the designer to optimize the design based on customer preference. For each of the selected responses, the payload length is selected as one of the factors. The methodology in Chapter Two will determine if the payload length is a critical factor, and if so, to what extent it impacts the design. Once the response curve is generated, the designer can perform trade-off studies to examine the impact of changing the payload length on the selected responses of cost, submerged displacement, length, submerged speed, and the OMOE. Prior to performing this analysis, this chapter will first review the methodology used by Hanson and Hunt to select their final concept design.

## **4.1 Hanson and Hunt**

When performing their concept exploration to select their final concept design, the design team was not aware of RSM. They selected their final concept design as follows: The sponsor of the project directed the design team to perform their trade-off analysis by varying the following four factors: submerged speed, test depth, diameter, and payload length. These parameters were selected to capture major design drivers (submerged speed and test depth), geometry (diameter), and the main useful characteristic for a modular design (payload length). Starting with a baseline design, 18 variants were used to accurately populate a design space. The variants were developed by deviating from the baseline design in each of the four factors.

The design team had originally planned on using the 18 variants to create a Pareto plot to help select the final concept design. It was at this time the design team learned of the capability of RSM and the statistical software JMP<sup>®</sup>. With the knowledge of RSM and the JMP<sup>®</sup> software, the design team wanted to recreate their variants using either the Box-Behnken or the Central Composite design (the factors were specified by the sponsor). Yet time constraints forced them to use the original 18 variants. On the other hand, the design team was able to use the JMP<sup>®</sup> software to their advantage. Each variant (factor levels and responses of interest) was entered in the JMP<sup>®</sup> program and, using standard least squares model fitting, the design team created a quadratic response surface as a function of the four factors. The design team then used the response surface to help select the final concept design.

In summary, the following observations are provided regarding the methodology used by the design team to select their final concept design:

- The factors were selected first, based on designer experience. The design team did not first pick the response of interest and then use the DOE methodology to determine the critical factors.
- With the selected factors, the design team did not use the Box-Behnken or the Central Composite design to develop the variants. Instead, they systematically varied the factors and used the statistical software package to create a curve fit (quadratic) as a function of the four factors.

The purpose of this section is to recreate the response surface using the Box-Behnken design. The analysis proceeded as follows:

- Use the same four factors: submerged speed, test depth, diameter, and payload length.
- The selected factor levels are shown in Table 6 (same as Hanson and Hunt).
- Use the Box-Behnken design to develop each design variant.<sup>2</sup> With four factors, the Box-Behnken design requires 27 variants to model the design space. Appendix A contains the design matrix and the responses.
- Select shaft horsepower, cost, and the transverse metacentric height (GMt) as the responses (this is a subset of the 19 responses selected by Hanson and Hunt).
- Using the Box-Behnken design, a response surface is created by modeling the surface with Equation (1) as a function of the four factors.

This analysis is only applying a portion of the methodology outlined in Chapter Two, specifically the application of the Box-Behnken design to create the response surface. But the factors selected were identified by designer experience, not by using the screening experiment to identify the critical factors. The results of the shaft horsepower,

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<sup>2</sup> The author initially attempted to use the Central Composite design, but the corner points were outside the limits of the parametric equations used in the modular submarine model.

cost and GMt response surface model will be analyzed. Then in the following section, the complete methodology outlined in Chapter Two will be applied to the cost response surface, resulting in an improved model than shown here.

Factor	-1	0	+1
Submerged speed (knots)	24	28	32
Test depth (ft)	500	800	1100
Diameter (ft)	38	40	42
Payload length (ft)	43	65.5	88

**Table 6: Cost Model Factor Levels**

#### **4.1.1 Shaft Horsepower Response Model**

A shaft horsepower response model is created as a function of the following factors: submerged speed, test depth, diameter, and payload length. The analysis of the fitted model is shown in Table 7 and 8 and Figure 9.

The Analysis of Variance, Table 7, summarizes the quality of the model fit to the actual shaft horsepower responses. The Total sum of squares (SS) is the sum of squared distances of each shaft horsepower response from the shaft horsepower sample mean. The Error SS is the sum of squared differences between the predicted model values and the actual values. This SS corresponds to the unexplained residual Error after fitting the regression model. The Model SS is then the Total SS minus the Error SS. If the selected factors accurately model the shaft horsepower response, then the Model SS will be much larger than the Error SS and is reflected in the F Ratio. The larger the F Ratio, the better the model fit.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	3166359451	226168532	101.9121
Error	12	26631012.4	2219251	Prob > F
C. Total	26	3192990464		<.0001

Table 7: Shaft Horsepower Response Model Analysis of Variance

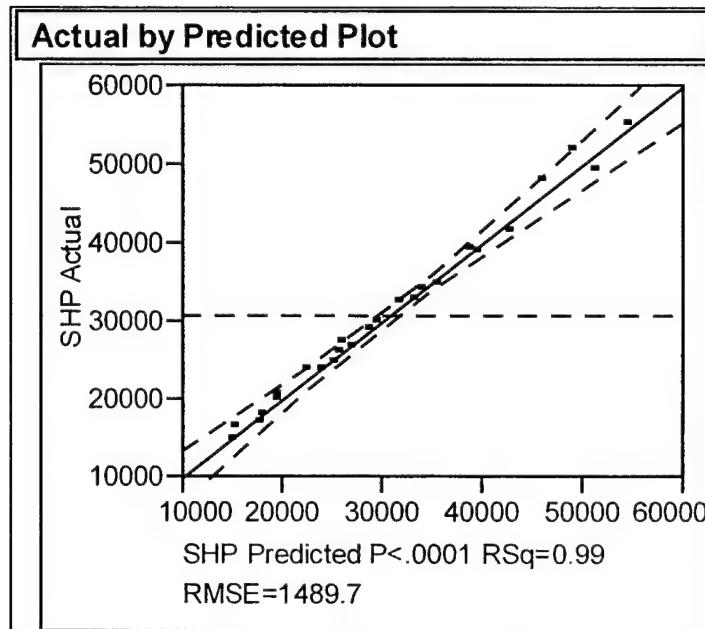


Figure 9: Shaft Horse Power Response Model Leverage Plot

Scaled Estimates					
Term	Scaled Estimate		Std Error	t Ratio	Prob> t
Intercept	28931.1		860.0874	33.64	<.0001
Diameter(38,42)&RS	866.68333		430.0437	2.02	0.0668
Depth(500,1100)&RS	3205.0192		430.0437	7.45	<.0001
Speed(24,32)&RS	14821.028		430.0437	34.46	<.0001
Payload(43,88)&RS	4911.1755		430.0437	11.42	<.0001
Diameter(38,42)*Depth(500,1100)	281.625		744.8575	0.38	0.7120
Diameter(38,42)*Speed(24,32)	665.075		744.8575	0.89	0.3895
Depth(500,1100)*Speed(24,32)	1043.2825		744.8575	1.40	0.1866
Diameter(38,42)*Payload(43,88)	284.15		744.8575	0.38	0.7095
Depth(500,1100)*Payload(43,88)	1441.75		744.8575	1.94	0.0768
Speed(24,32)*Payload(43,88)	2652.2265		744.8575	3.56	0.0039
Diameter(38,42)*Diameter(38,42)	639.81517		645.0656	0.99	0.3408
Depth(500,1100)*Depth(500,1100)	204.94392		645.0656	0.32	0.7562
Speed(24,32)*Speed(24,32)	3269.6822		645.0656	5.07	0.0003
Payload(43,88)*Payload(43,88)	106.32842		645.0656	0.16	0.8718

Table 8: Shaft Horse Power Response Model Coefficients

The F Ratio is a statistical tool to test the hypothesis that all coefficients in Equation (1) are zero. If the hypothesis is not true, i.e. at least one coefficient is non-zero, then the F Ratio will be large. The "Prob > F" in Table 7 is the probability of obtaining a greater F Ratio by chance alone if the specified model fits no better than the overall response mean. Significance probabilities of 0.05 or less are often considered evidence that there is at least one significant regression factor in the model. Since Table 7 has "Prob > F" as less than 0.001, the model is an excellent predictor of the required shaft horsepower.

Another method to examine the quality of the model fit is the leverage plot, Figure 9. By examining the graph, the designer can decide if the model predicted values (middle solid line) is a better fit than the shaft horsepower sample mean (dashed horizontal line). The dashed lines on each side of the model predicted values are the 95 percent confidence curves. These indicate whether the F Test is significant at the 5 percent level by showing a confidence region for the model predicted values. If the 95 percent confidence curves cross the shaft horsepower sample mean, then the model is significant; if the curves do not cross, then it is not significant at the 5 percent level. Examination of Figure 9 illustrates that the model is significant.

The RSq in Figure 9 estimates the proportion of the variation in the shaft horsepower response around the mean that can be attributed to terms in the model rather than to random error. An RSq of 1 occurs with there is a perfect fit (all errors are zero). An RSq of 0 means that the model fit predicts the response no better than the overall mean response. For the shaft horsepower response, Figure 9 displays an RSq of 0.99, therefore the shaft horsepower response model is an excellent fit.



The RMSE (Root Mean Square Error) in Figure 9 estimates the standard deviation of the random error. It is the square root of the mean square for error in the corresponding Analysis of Variance table.

Table 8 shows the sample estimates of the coefficients for Equation (1). The standard error is an estimate of the standard deviation for each coefficient. The t Ratio and “Prob > |t|,” similar to the F Ratio, test for the hypothesis that each coefficient is zero. A very large t Ratio is an indicator that the true coefficient might not be zero. Therefore, the “Prob > |t|” is the probability of generating an even greater t statistic, given that the coefficient is zero. Probabilities less than 0.05 are often considered as significant evidence that the coefficient is not zero. Reviewing Table 8, the submarine diameter has a “Prob > |t|” of 0.0668, indicating that the diameter is not statistically significant, i.e. it has very little impact on the shaft horsepower response. On the other hand, test depth, submerged speed and the payload length are statistically significant and do impact the shaft horsepower response. Also note that the interaction between submerged speed and payload length is statistically significant.

The analysis of the results in Table 7 and 8 and Figure 9 is summarized as follows: The quadratic model, with the coefficients defined in Table 8, is statistically significant. The model can accurately predict the required shaft horsepower as a function of the four factors, with an estimated standard deviation of 1489.7 shaft horsepower. Examining Table 8, the graphical display shows that the submerged speed is a large driver for the shaft horsepower requirement followed by payload length and test depth.

On the other hand, the standard deviation of 1489.7 shaft horsepower is quite large. But upon analysis of the modular submarine model, there are three uncontrolled variances that effect the accuracy of the model fit. They are as follows:

- Propulsive coefficient: this value was not held constant and is modeled as a function of the size and shape of the hull.
- Appendage area: The model increases the appendage area as the size of the submarine increases.
- Weight-limiting versus volume-limiting design: Some designs are weight limiting; others are volume limiting.

#### **4.1.2 Cost Response Model**

A cost response model is created as a function of the following factors: submerged speed, test depth, diameter, and payload length. The analysis of the fitted model is shown in Table 9 and 10 and Figure 10. As shown in Table 9, the “Prob > F” is less than 0.001, indicating the model is statistically significant and is an excellent predictor of cost (assuming the cost model is correct) with a standard deviation of \$65.804 million (the units in Figure 10 is million of dollars). Examining the coefficients in Table 10, the big cost driver is the payload length, followed by the submerged speed and test depth.

This chapter will redo this analysis using the complete methodology outlined in Chapter Two. The analysis will produce a better model with a smaller standard deviation.

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	3169393.6	226385	52.2803
Error	12	51962.6	4330	Prob > F
C. Total	26	3221356.2		<.0001

Table 9: Cost Response Model Analysis of Variance

### Actual by Predicted Plot

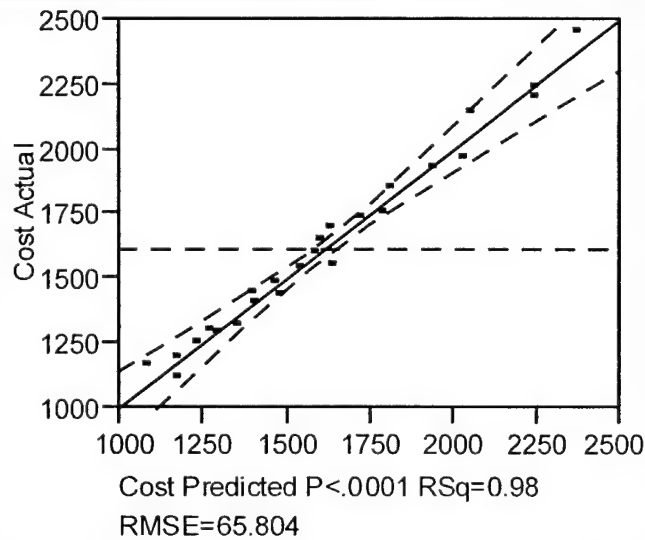


Figure 10: Cost Response Model Leverage Plot

### Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob> t
Intercept	1540.32	37.99219	40.54	<.0001
Diameter(38,42)&RS	89.265	18.99609	4.70	0.0005
Depth(500,1100)&RS	243.495	18.99609	12.82	<.0001
Speed(24,32)&RS	261.03417	18.99609	13.74	<.0001
Payload(43,88)&RS	337.69417	18.99609	17.78	<.0001
Diameter(38,42)*Depth(500,1100)	22.1225	32.9022	0.67	0.5141
Diameter(38,42)*Speed(24,32)	32.095	32.9022	0.98	0.3486
Depth(500,1100)*Speed(24,32)	60.915	32.9022	1.85	0.0889
Diameter(38,42)*Payload(43,88)	30.0625	32.9022	0.91	0.3789
Depth(500,1100)*Payload(43,88)	87.7075	32.9022	2.67	0.0206
Speed(24,32)*Payload(43,88)	108.3925	32.9022	3.29	0.0064
Diameter(38,42)*Diameter(38,42)	18.374167	28.49414	0.64	0.5312
Depth(500,1100)*Depth(500,1100)	29.346667	28.49414	1.03	0.3234
Speed(24,32)*Speed(24,32)	113.80792	28.49414	3.99	0.0018
Payload(43,88)*Payload(43,88)	13.257917	28.49414	0.47	0.6501

Table 10: Cost Response Model Coefficients

### 4.1.3 GMt Response Model

A GMt response model is created as a function of the following factors: submerged speed, test depth, diameter, and payload length. The analysis of the model is shown in Table 11 and Figure 11. The Analysis of Variance has “Prob > F” at 0.0496, indicating that the fitted model is marginal as a predictor. Figure 11 also indicates a poor fit. Therefore, there are uncontrolled variances that have not been accounted for, which should be either held constant or modeled in Equation (1). To improve upon the model fit, the designer would need to examine all possible factors that could effect GMt, and then perform the screening experiment to determine the critical factors.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	0.01138241	0.000813	2.6428
Error	12	0.00369167	0.000308	Prob > F
C. Total	26	0.01507407		0.0496

Table 11: GMt Response Model Analysis of Variance

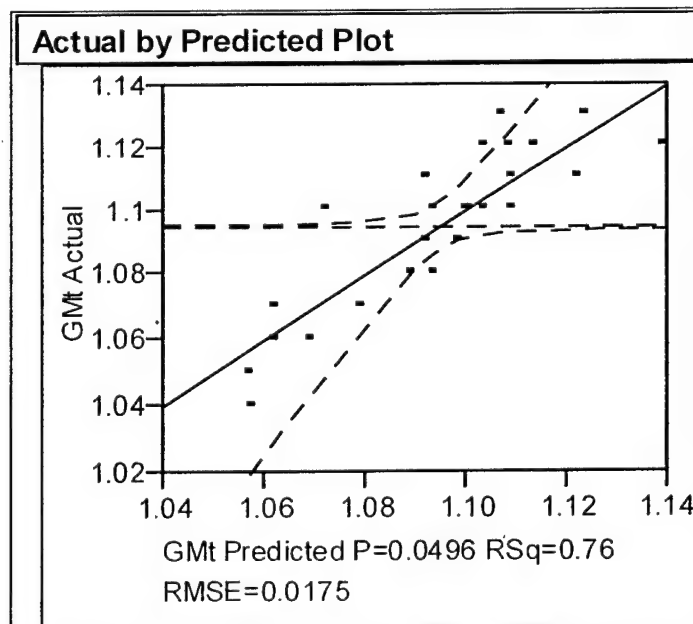


Figure 11: GMt Response Model Leverage Plot

#### 4.1.4 Design Space Analysis

Using the JMP® software, the designer can use the contour profiler (Figure 12), which is an interactive contour profiling tool used to optimize the response surfaces graphically. Figure 12 is used to examine dynamically how the four factors impact the response. Two of the four factors are selected as the x-axis (test depth) and the y-axis (submerged speed). The other two are fixed: diameter at 40 ft and the payload length at 64.0 ft. In this example, the cost response contour is set at an upper limit of \$2.0 billion. Therefore, if the designer is given a cost constraint of \$2.0 billion, the shaded portion of the graph represents a portion of the design space that exceeds the \$2.0 billion limit thereby reducing the feasible design space. The shaft horsepower contour is set at 30,000. By visually examining the graph, and by keeping the shaft horsepower constant, the submarine will lose about 1.5 knots as the test depth increases from 500 to 1100 ft.

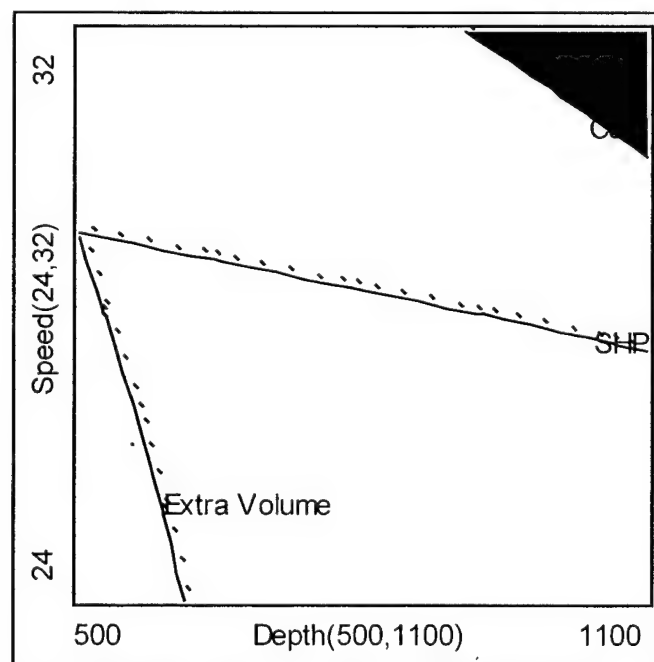


Figure 12: Hanson and Hunt Contour Profiler

The extra volume response in Figure 12 represents the additional volume added to the design when the submarine is weight limited. As shown in this figure, the extra volume contour is set at zero and represents the transition from a volume-limited to a weight-limited design. This indicates that the modular submarine module is, for the most part, a weight-limited design (in the specified design space).

Once created, the contour profiler allows the designer to examine an infinite number of designs within the design space. The ability to dynamically change the design factors and examine the impact on the response is a powerful tool. The designer can now perform trade-off studies instantly and determine the impact of changing a design factor. Just as important, the designer can input the design constraints (i.e. a cost limit as in Figure 12) and examine the potential design variants in the remaining feasible design space, if any. The importance of using the contour profiler for trade-off studies will be illustrated later in the chapter.

## ***4.2 Cost Response Model***

In Section 4.1.2, a cost response model is generated as a function of the factors submerged speed, test depth, diameter, and payload length. Figure 10 illustrates the results of the model fit:  $RSq$  of 0.98 and a standard deviation of \$65.804 million. As described earlier, the factors were specified (a screening experiment was not conducted), and a Box-Behnken design is used to create the response surface. This section will now perform the methodology outlined in Chapter Two, and the analysis will result in an improved cost model.

### **4.2.1 Factor Selection**

Since the modular submarine model uses weight-based Cost-Estimating Ratios (CER), the key to identification of the factors are those that drive the different weight groups. The selected factors for the cost response are as follows:

- Group 1 (hull structure): test depth and payload length
- Group 2 (propulsion machinery): shaft horsepower requirement
- Group 3 (electric plant): shaft horsepower requirement
- Group 4 (communications and control): held constant
- Group 5 (auxiliary systems): held constant
- Group 6 (outfit and furnishings): held constant
- Group 7 (armament or payload): payload length and diameter

Therefore, the selected factors are test depth, payload length, shaft horsepower, and diameter. These factors are identical to Section 4.1.2, except for shaft horsepower. This section uses shaft horsepower which is directly related to the Group 2 and 3 weights. On the other hand, Section 4.1.2 used submerged speed, which is a function of many design parameters, including shaft horsepower. The selection of shaft horsepower rather than submerged speed will improve the model fit.

### **4.2.2 Screening Experiment**

A full factorial design matrix is selected to examine the main effects and factor interactions and to determine if they are statistically significant. The full factorial with four factors only requires 16 variants. The selected factor levels are shown in Table 12, and Appendix B contains the full factorial design matrix annotated with the factor levels and the cost response. When picking the factor levels, one should pick in the design

range of interest. In this case, the levels are selected to produce a weight-limiting concept design within the entire design space.

Factor	-1	+1
Test Depth (ft)	700	1100
Payload length (ft)	65	85
Shaft horsepower	20000	50000
Diameter	38	42

**Table 12: Cost Response Screening Experiment Factor Levels**

The results of the screening experiment are shown in Figure 13 and Table 13. The prediction profiler, Figure 13, is a graph of the predicted response (on the y-axis) as one factor (on the x-axis) is changed while the others are held constant at their current levels. Since a two-level screening experiment is conducted, a linear function is obtained. The steepness, or slope, is a measure of the factor effect on the response. By visual inspection of Figure 13, the shaft horsepower has the largest impact on the cost, while the diameter has the least impact. Once again, the results will vary if the range for the factors change, so it is critical to carefully select the proper range for each factor. The shaft horsepower range is from 20,000 to 50,000, and this large range explains its impact on the cost response. On the other hand, the diameter has a small range from 38 to 42 ft and therefore has a minimal impact on the response, as compared to the other factors. The key is to select the range for each factor to correspond to the design space of interest.



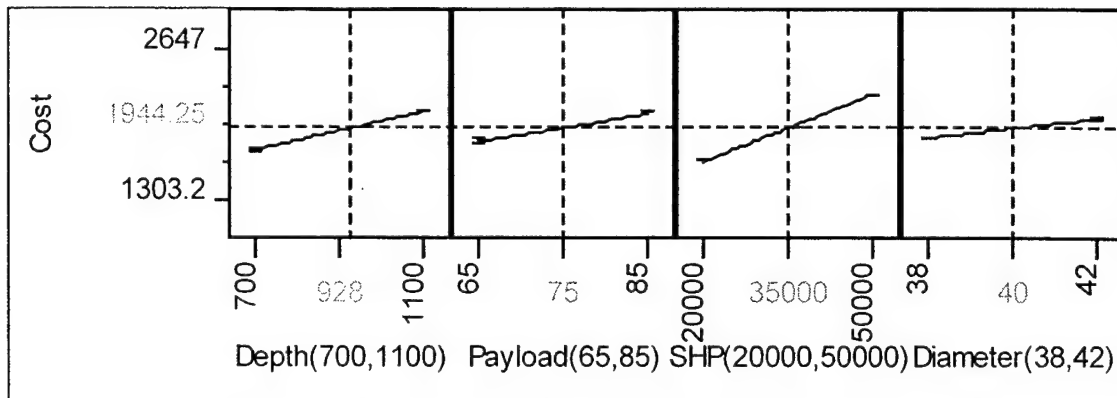


Figure 13: Cost (Screening Experiment) Prediction Profiler

Scaled Estimates						
Term	Scaled Estimate			Std Error	t Ratio	Prob> t
Intercept	1920.5994			0.60383	3180.70	<.0001
Depth(700,1100)	168.93563			0.60383	279.77	<.0001
Payload(65,85)	120.22312			0.60383	199.10	<.0001
SHP(20000,50000)	294.62313			0.60383	487.92	<.0001
Diameter(38,42)	86.986875			0.60383	144.06	<.0001
Depth(700,1100)*Payload(65,85)	14.006875			0.60383	23.20	<.0001
Depth(700,1100)*SHP(20000,50000)	19.059375			0.60383	31.56	<.0001
Payload(65,85)*SHP(20000,50000)	0.066875			0.60383	0.11	0.9161
Depth(700,1100)*Diameter(38,42)	9.973125			0.60383	16.52	<.0001
Payload(65,85)*Diameter(38,42)	11.475625			0.60383	19.00	<.0001
SHP(20000,50000)*Diameter(38,42)	0.045625			0.60383	0.08	0.9427

Table 13: Cost (Screening Experiment) Results

The t Test results for the screening experiment are shown in Table 13. Examining the “Prob > |t|” column, all factor and two-way interactions are statistically significant, except for the two-way interactions (payload length \* shaft horsepower) and (shaft horsepower \* diameter). All four factors will be used to develop the cost response surface.

### 4.2.3 Response Surface

A cost response surface is created using the Central Composite design. With four factors, the selected Central Composite design requires 26 variants to accurately model the design space. To capture the quadratic curvature, three levels per factor (see Table 14) are used. Appendix C contains the design matrix annotated with the factor levels and the cost response. The results are shown in Table 15 and 16, and Figures 14 and 15.

Factor	-1	0	+1
Test depth (ft)	700	900	1100
Payload length (ft)	65	76.5	88
Shaft horsepower	20000	35000	50000
Diameter (ft)	38	40	42

Table 14: Cost Response Curve Factor Levels

The results of the cost model fit are excellent. The Analysis of Variance, Table 15, has a “Prob > F” of less than 0.0001 and the RSq, figure 14, is 1.00. With a \$3.4762 million standard deviation, the results of this section are much improved compared to the Hanson and Hunt \$65.804 million standard deviation. Table 16 and Figure 15 show the effect of each factor and their interactions on the cost response surface model. Note that the curvature in Figure 15 is due to using three-level factors to create the quadratic response surface. Examination of Table 16 shows that the main effects are the drivers for cost. Although the factor interactions are statistically significant (except for payload length\* shaft horsepower and diameter \* shaft horsepower, same as the screening experiment), they do not have the impact that the main effects have on the cost response model.

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	2650424.1	189316	15667.01
Error	11	132.9	12	Prob > F
C. Total	25	2650557.1		<.0001

Table 15: Cost Response Model Analysis of Variance

### Actual by Predicted Plot

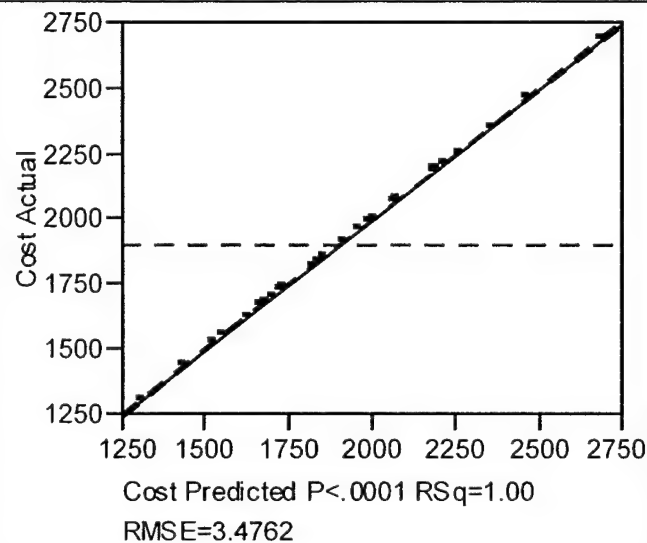


Figure 14: Cost Response Model Leverage Plot

### Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob> t
Intercept	1820.7287	1.377996	1321.29	<.0001
Depth(700,1100)&RS	170.26778	0.81934	207.81	<.0001
Payload(65,88)&RS	138.04722	0.81934	168.49	<.0001
SHP(20000,50000)&RS	294.42889	0.81934	359.35	<.0001
Diameter(38,42)&RS	88.587778	0.81934	108.12	<.0001
Depth(700,1100)*Payload(65,88)	16.085625	0.869042	18.51	<.0001
Depth(700,1100)*SHP(20000,50000)	19.058125	0.869042	21.93	<.0001
Payload(65,88)*SHP(20000,50000)	0.063125	0.869042	0.07	0.9434
Depth(700,1100)*Diameter(38,42)	10.171875	0.869042	11.70	<.0001
Payload(65,88)*Diameter(38,42)	13.196875	0.869042	15.19	<.0001
SHP(20000,50000)*Diameter(38,42)	0.051875	0.869042	0.06	0.9535
Depth(700,1100)*Depth(700,1100)	15.916714	2.172191	7.33	<.0001
Payload(65,88)*Payload(65,88)	0.1417143	2.172191	0.07	0.9492
SHP(20000,50000)*SHP(20000,50000)	99.496714	2.172191	45.80	<.0001
Diameter(38,42)*Diameter(38,42)	2.3567143	2.172191	1.08	0.3012

Table 16: Cost Response Model Coefficients

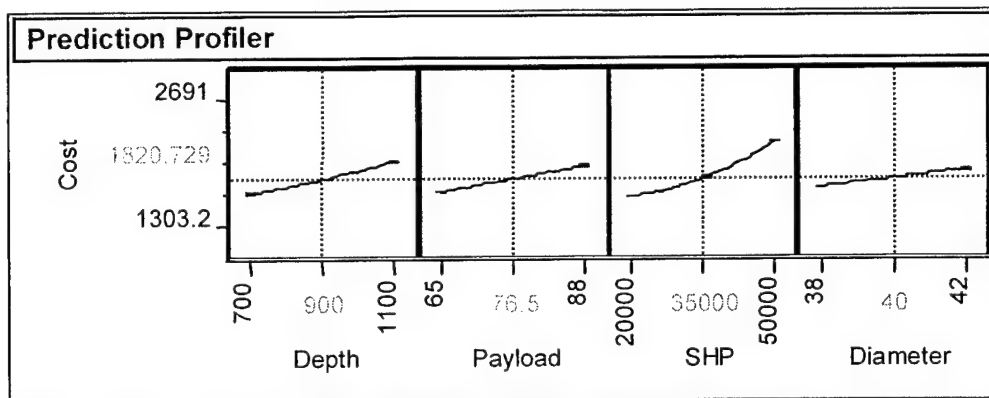


Figure 15: Cost (Response Surface) Prediction Profiler

### 4.3 Submerged Displacement Response Model

Since the cost response model is based on weight Cost Estimating Ratios, the factors for a submerged displacement response model are identical to the cost response model as long as the concept designs are weight limited. Using the same Central Composite design matrix (see Appendix C), a submerged displacement response model is generated. The results are displayed in Tables 17 and 18, and Figure 16.

Similar to the cost response model, the submerged displacement response model is an excellent fit. The Analysis of Variance, Table 17, has a "Prob > F" of less than 0.0001 and the RSq, Figure 16, is 1.00. The model can accurately predict the submerged displacement with a standard deviation of 26.958 ltms. The model coefficients, Table 18, have the same relative magnitude as the coefficients for the cost response surface model. The main effects of test depth, payload length, shaft horsepower, and diameter are the key drivers for the submerged displacement.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	103892891	7420921	10211.44
Error	11	7994	727	Prob > F
C. Total	25	103900885		<.0001

Table 17: Submerged Displacement Response Model Analysis of Variance

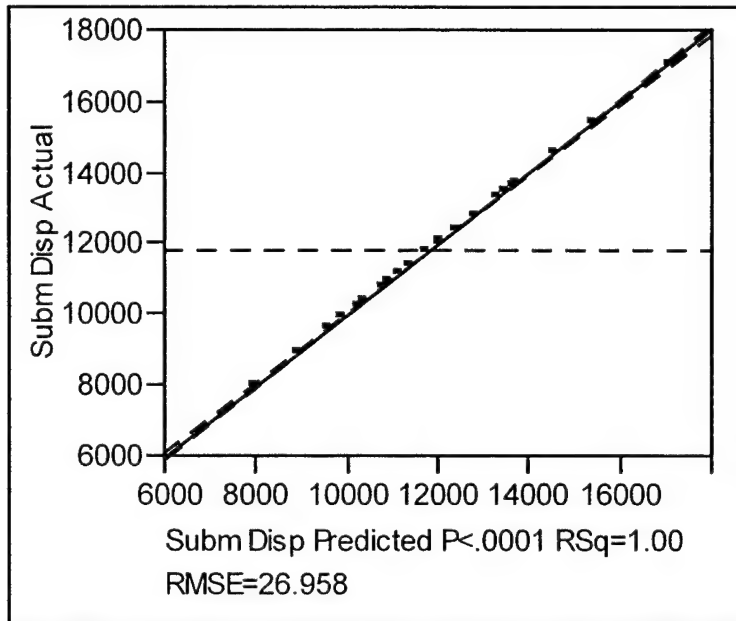


Figure 16: Submerged Displacement Response Model Leverage Plot

Scaled Estimates					
Term	Scaled Estimate		Std Error	t Ratio	Prob> t
Intercept	11398.227		10.68643	1066.61	<.0001
Depth(700,1100)&RS	1310.7556		6.354028	206.29	<.0001
Payload(65,88)&RS	1024.7611		6.354028	161.28	<.0001
SHP(20000,50000)&RS	1558.5611		6.354028	245.29	<.0001
Diameter(38,42)&RS	639.09444		6.354028	100.58	<.0001
Depth(700,1100)*Payload(65,88)	124.20625		6.739465	18.43	<.0001
Depth(700,1100)*SHP(20000,50000)	146.10625		6.739465	21.68	<.0001
Payload(65,88)*SHP(20000,50000) *	0.25625		6.739465	0.04	0.9704
Depth(700,1100)*Diameter(38,42)	78.51875		6.739465	11.65	<.0001
Payload(65,88)*Diameter(38,42)	94.61875		6.739465	14.04	<.0001
SHP(20000,50000)*Diameter(38,42)	0.29375		6.739465	0.04	0.9660
Depth(700,1100)*Depth(700,1100)	122.91381		16.84545	7.30	<.0001
Payload(65,88)*Payload(65,88)	0.6638095		16.84545	0.04	0.9693
SHP(20000,50000)*SHP(20000,50000)	526.16381		16.84545	31.23	<.0001
Diameter(38,42)*Diameter(38,42)	17.26381		16.84545	1.02	0.3275

Table 18: Submerged Displacement Response Model Coefficients

#### ***4.4 Length Response Model***

Since the modular submarine model is a weight-limiting design (in the design space of interest), the factors for the length response model are identical to the submerged displacement model. Using the same Central Composite design matrix (see Appendix C), a length response model is generated. The results are displayed in Tables 19 and 20, and Figure 17. The length response model can accurately predict the length with a standard deviation of 1.0396 ft.

#### ***4.5 Submerged Speed Response Model***

This section will develop the response surface for the submerged speed. Since there are numerous factors that can impact speed, the application of the methodology outlined in Chapter Two will aid the designer in determining the critical factors and which critical factors have the greatest impact on the submerged speed. The designer can then use the subset (which have the greatest impact on the submerged speed) of critical factors for further study. This will be illustrated in this section.

##### **4.5.1 Factor Selection**

The first step is to determine the potential factors that may have an impact on the submerged speed. The factors can be categorized as follows: hull form, hull size, and the installed propulsion system.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	88463.630	6318.83	5283.954
Error	11	13.154	1.20	Prob > F
C. Total	25	88476.785		<.0001

Table 19: Length Response Model Analysis of Variance

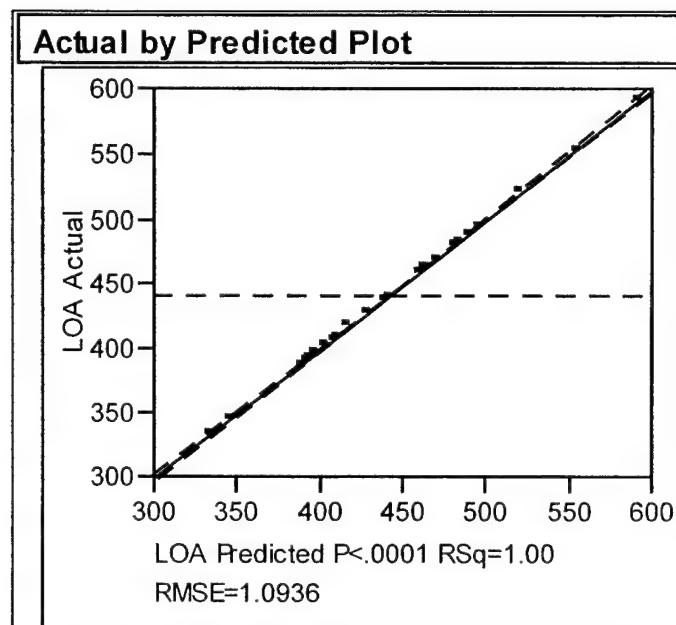


Figure 17: Length Response Model Leverage Plot

Scaled Estimates					
Term	Scaled Estimate		Std Error	t Ratio	Prob> t
Intercept	427.84857		0.433497	986.97	<.0001
Depth(700,1100)&RS	38.477778		0.257752	149.28	<.0001
Payload(65,88)&RS	31.722222		0.257752	123.07	<.0001
SHP(20000,50000)&RS	46		0.257752	178.47	<.0001
Diameter(38,42)&RS	-12.35556		0.257752	-47.94	<.0001
Depth(700,1100)*Payload(65,88)	3.6375		0.273388	13.31	<.0001
Depth(700,1100)*SHP(20000,50000)	4.325		0.273388	15.82	<.0001
Payload(65,88)*SHP(20000,50000)	-0.0125		0.273388	-0.05	0.9644
Depth(700,1100)*Diameter(38,42)	-1.5625		0.273388	-5.72	0.0001
Payload(65,88)*Diameter(38,42)	-0.225		0.273388	-0.82	0.4280
SHP(20000,50000)*Diameter(38,42)	-4.5875		0.273388	-16.78	<.0001
Depth(700,1100)*Depth(700,1100)	3.6685714		0.683339	5.37	0.0002
Payload(65,88)*Payload(65,88)	0.0685714		0.683339	0.10	0.9219
SHP(20000,50000)*SHP(20000,50000)	15.468571		0.683339	22.64	<.0001
Diameter(38,42)*Diameter(38,42)	1.1685714		0.683339	1.71	0.1153

Table 20: Length Response Model Coefficients

For a body of revolution, the hull form is determined by the length, diameter, and two form factors (entrance and run form coefficient) [12]. Therefore, the selected factors are the diameter, and the entrance and run form coefficients. Length is not selected as a factor because it must vary (a response) to account for varying submarine sizes. In addition, attached to the hull will be the sail and numerous appendages. This can be modeled by representing the sail and appendages with an equivalent area. Therefore, the additional factors are the sail and appendage areas.

The hull size is equivalent to the submerged displacement. Therefore, from section 4.3, the factors are test depth, installed shaft horsepower, payload length and diameter.

Finally, the propulsion system can be defined by numerous factors. Yet within the modular submarine model, the installed shaft horsepower and the propulsive coefficient (PC) determine the installed propulsion system.

In summary, Table 21 lists the selected factors for the screening experiment.

Factor	-1	+1
Diameter	38	42
Forward form coefficient	1.75	3.5
Aft form coefficient	1.75	4.0
Shaft horsepower	45000	50000
PC	0.79	0.85
Sail area	285	575
Appendage area	2000	3500
Depth	700	1100
Payload length	65	88

**Table 21: Submerged Speed Screening Experiment Factor Levels**



#### 4.5.2 Screening Experiment

The rationale for the factor levels is as follows: If the current concept design has a PC of 0.79 and the designer is informed that a PC of 0.85 is possible, but at a specified cost, then the range is 0.79 to 0.85, as shown in Table 21. This will allow the designer to see if PC is a critical factor or not. If it is, then the designer can further evaluate the possibility of increasing the PC value at the additional cost by using the response surface. If, on the other hand, it is determined that PC is not a critical factor, then the designer can spend his effort and money on the other factors that do have an impact. The same rationale is applied to the remaining factors, except for shaft horsepower. This is a critical factor for submerged speed and has a dramatic impact. To examine the relative effects of the other factors, the shaft horsepower levels are chosen to reduce the impact of this factor on the response. Therefore, the shaft horsepower levels are from 45,000 to 50,000. At this level, the effect of shaft horsepower is much less than going from 0 to 5,000.

To perform the screening experiment, a fractional factorial design matrix, resolution IV, is selected. The resolution IV design will ensure the main effects are not confounded with the two-way interactions, but the two-way interactions will be confounded with other two-way interactions. A fractional factorial with nine factors requires 32 variants, and Appendix D contains the design matrix annotated with the factor levels and the submerged speed response. The results are displayed in Figure 18 and 19, and Table 22.

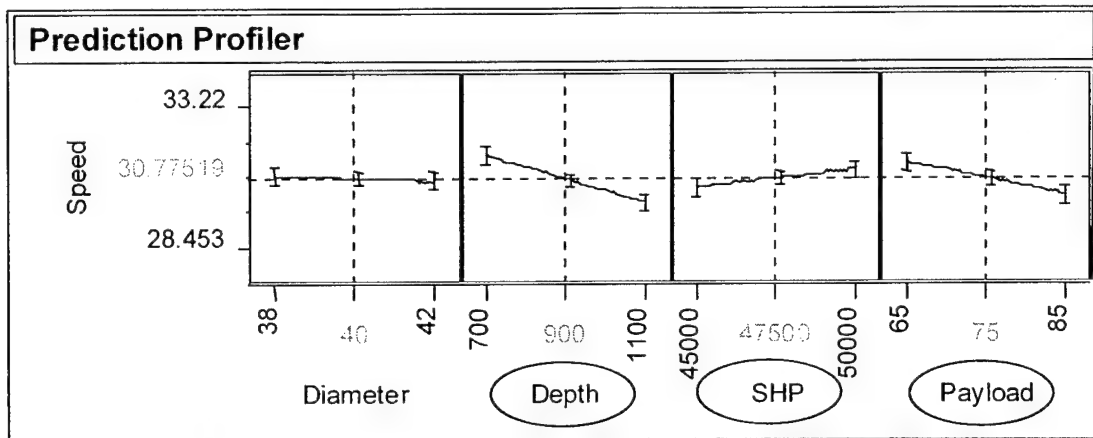


Figure 18: Submerged Speed Screening Experiment Prediction Profiler (1/2)

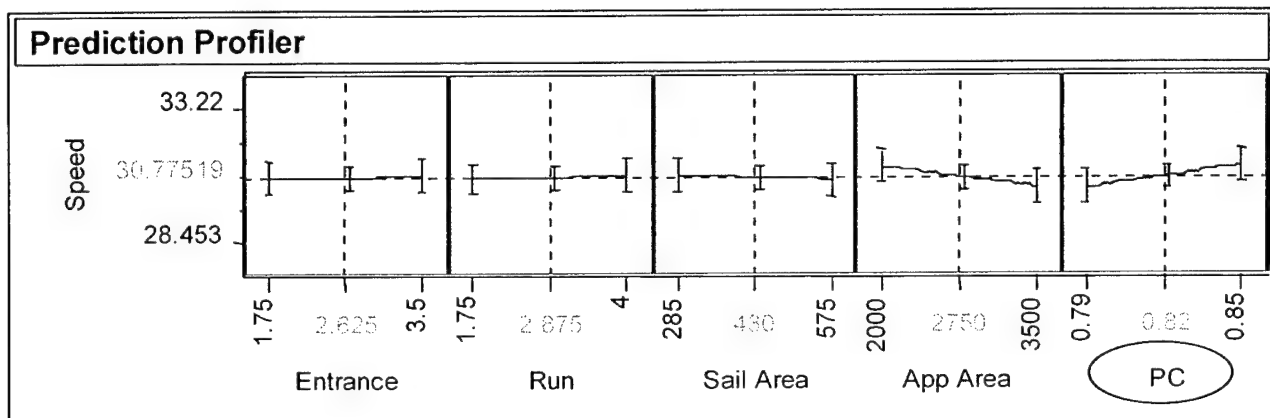


Figure 19: Submerged Speed Screening Experiment Prediction Profiler (2/2)

The prediction profilers, Figure 18 and 19, show that the main effects of test depth, shaft horsepower, payload length, appendage area, and PC have the largest impact on the submerged speed. Based on this visual examination, the following factors have been selected to create the response surface: test depth, shaft horsepower, payload length, and PC. The appendage area is not selected since the size of the control surfaces is determined by other factors. The other factors not selected for creating the response surface are all statically significant, as shown in Table 22, but they do not have the impact on the submerged speed as the selected factors. This example illustrates the

importance of the screening experiment; the designer can focus on the design parameters that are statistically significant and have the largest impact within the design space of interest.

Table 22 shows the results of the t Test for the main effects and all two-way interactions (not shown are the three-way interactions, which are not statistically significant). Even though some of the two-way interactions are statistically significant, none have the same impact on the response as the main effects.

Scaled Estimates					
Term	Scaled Estimate		Std Error	t Ratio	Prob> t
Intercept	30.775187		0.000375	82067.19	<.0001
Diameter(38,42)	-0.085562		0.000375	-228.17	0.0028
Depth(700,1100)	-0.80625		0.000375	-2150.00	0.0003
SHP(45000,50000)	0.28725		0.000375	766.00	0.0008
PL(65,85)	-0.541438		0.000375	-1443.83	0.0004
Entrance(1.75,3.5)	0.0379375		0.000375	101.17	0.0063
Run(1.75,4)	0.06625		0.000375	176.67	0.0036
Sail Area(285,575)	-0.080625		0.000375	-215.00	0.0030
App Area(2000,3500)	-0.349187		0.000375	-931.17	0.0007
PC(0.79,0.85)	0.3881875		0.000375	1035.17	0.0006
Diameter(38,42)*Depth(700,1100)	0.003		0.000375	8.00	0.0792
Diameter(38,42)*SHP(45000,50000)	0.013		0.000375	34.67	0.0184
Diameter(38,42)*PL(65,85)	-0.020063		0.000375	-53.50	0.0119
Diameter(38,42)*Entrance(1.75,3.5)	0.0034375		0.000375	9.17	0.0692
Diameter(38,42)*Run(1.75,4)	0.006125		0.000375	16.33	0.0389
Diameter(38,42)*Sail Area(285,575)	0.00025		0.000375	0.67	0.6257
Diameter(38,42)*App Area(2000,3500)	0.0033125		0.000375	8.83	0.0718
Diameter(38,42)*PC(0.79,0.85)	-0.000188		0.000375	-0.50	0.7048
Depth(700,1100)*SHP(45000,50000)	-0.017437		0.000375	-46.50	0.0137
Depth(700,1100)*PL(65,85)	0.0015		0.000375	4.00	0.1560
Depth(700,1100)*Entrance(1.75,3.5)	-0.0035		0.000375	-9.33	0.0680
Depth(700,1100)*Run(1.75,4)	-0.005062		0.000375	-13.50	0.0471
Depth(700,1100)*Sail Area(285,575)	0.0021875		0.000375	5.83	0.1081
Depth(700,1100)*App Area(2000,3500)	0.0335		0.000375	89.33	0.0071
Depth(700,1100)*PC(0.79,0.85)	0.00725		0.000375	19.33	0.0329
SHP(45000,50000)*Entrance(1.75,3.5)	-0.0075		0.000375	-20.00	0.0318
SHP(45000,50000)*Run(1.75,4)	-0.004437		0.000375	-11.83	0.0537
SHP(45000,50000)*Sail Area(285,575)	0.0258125		0.000375	68.83	0.0092
SHP(45000,50000)*App Area(2000,3500)	0.0115		0.000375	30.67	0.0208
Entrance(1.75,3.5)*Sail Area(285,575)	-0.005125		0.000375	-13.67	0.0465
Entrance(1.75,3.5)*App Area(2000,3500)	-0.003687		0.000375	-9.83	0.0645

Table 22: Submerged Speed Screening Experiment Results

### 4.5.3 Response Surface

A submerged speed response surface is created using the Central Composite design. With four factors, the selected Central Composite design requires 26 variants to accurately model the design space. To capture the quadratic curvature, three levels per factor (see Table 23) are used. Appendix E contains the design matrix annotated with the factor levels and the cost response. The results are shown in Table 24 and 25, and Figure 20.

Factor	-1	0	+1
Test depth (ft)	700	900	1100
Payload length (ft)	65	76.5	88
PC	0.79	0.82	0.85
Shaft horsepower	20000	35000	50000

**Table 23: Submerged Speed Model Factor Levels**

The results of the submerged speed model fit are excellent. The Analysis of Variance, Table 24, has a “Prob > F” of less than 0.0001 and the RSq, figure 20, is 1.00. This submerged speed model is an excellent predictor of speed with a standard deviation of 0.0263 knots. As seen in Table 25, the main effects are the drivers for submerged speed. Although some of the factor interactions are statistically significant, they do not have the same impact on submerged speed as the main effects.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	228.28108	16.3058	23486.4
Error	11	0.00764	0.0007	Prob > F
C. Total	25	228.28872		<.0001

Table 24: Submerged Speed Response Model Analysis of Variance

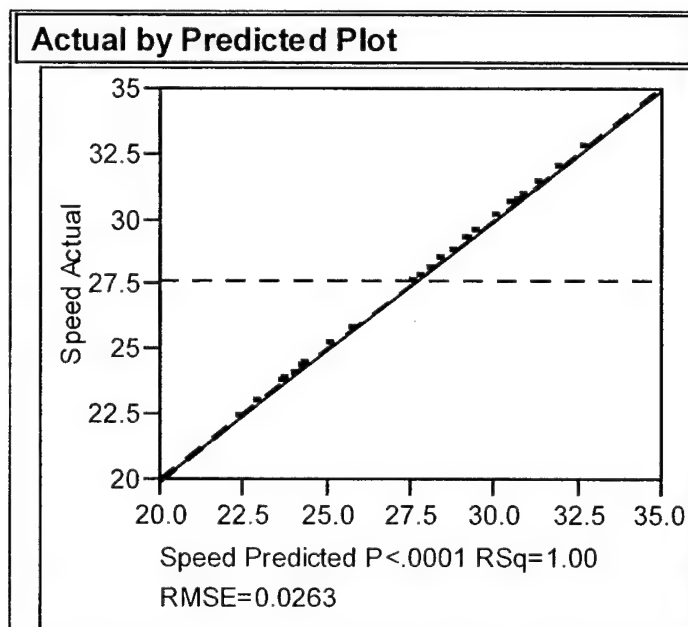


Figure 20: Submerged Speed Response Model Leverage Plot

Scaled Estimates				
Term	Scaled Estimate		Std Error	t Ratio Prob> t
Intercept	28.451786		0.010445	2723.95 <.0001
Depth(700,1100)&RS	-0.817111		0.00621	-131.57 <.0001
Payload(65,88)&RS	-0.675		0.00621	-108.69 <.0001
PC(0.79,0.85)&RS	0.3468889		0.00621	55.86 <.0001
SHP(20000,50000)&RS	3.33		0.00621	536.19 <.0001
Depth(700,1100)*Payload(65,88)	0.0068125		0.006587	1.03 0.3232
Depth(700,1100)*PC(0.79,0.85)	-0.010313		0.006587	-1.57 0.1458
Payload(65,88)*PC(0.79,0.85)	-0.008563		0.006587	-1.30 0.2202
Depth(700,1100)*SHP(20000,50000)	-0.097188		0.006587	-14.75 <.0001
Payload(65,88)*SHP(20000,50000)	-0.005187		0.006587	-0.79 0.4476
PC(0.79,0.85)*SHP(20000,50000)	0.0419375		0.006587	6.37 <.0001
Depth(700,1100)*Depth(700,1100)	-0.028548		0.016465	-1.73 0.1108
Payload(65,88)*Payload(65,88)	0.0384524		0.016465	2.34 0.0395
PC(0.79,0.85)*PC(0.79,0.85)	-0.004548		0.016465	-0.28 0.7875
SHP(20000,50000)*SHP(20000,50000)	-1.053548		0.016465	-63.99 <.0001

Table 25: Submerged Speed Response Model Coefficients

With the submerged speed response surface created, the designer can now easily perform trade-off studies. Using the contour profiler in JMP®, Figure 21, the designer can dynamically examine an infinite number of feasible solutions. With the contour profiler the designer can change the four factors (shaft horsepower, test depth, payload length, and PC) and examine their impact on the submerged speed. For instance, if the speed requirement is 28.0 knots, the designer, via the contour profiler, can easily determine the shaft horsepower requirements for any combination of payload length, test depth, and PC. In this figure, an additional 7,000 shaft horsepower is required to change test depth from 700 to 1100 ft and maintain the submerged speed at 28.0 knots. If the required shaft horsepower at 1100 ft is too high, then the designer can reduce the payload length or increase PC to reduce the required shaft horsepower to the required level.

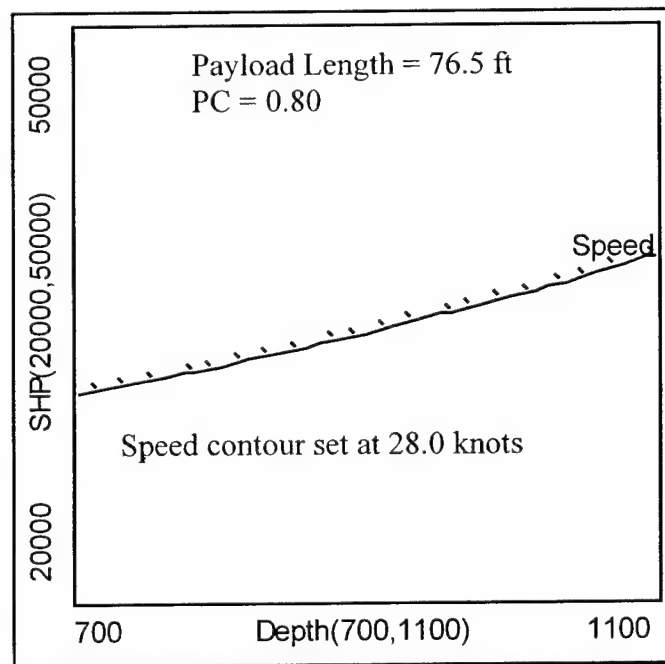


Figure 21: Submerged Speed Contour Profiler

## 4.6 OMOE Response Model

One method to optimize a concept design and perform trade-off studies is via the Pareto plot. To apply RSM, the designer will need to develop a response surface for both the cost and OMOE. In section 4.2 a response curve is generated for cost. This section will examine the development of an OMOE response surface.

Starting with an OMOE, the designer needs to reduce the OMOE to a function of factors. Using the OMOE from Hanson and Hunt, we have

$$OMO E = fn(test\ depth) + fn(payload\ length) + fn(submerged\ speed). \quad (4)$$

In Section 4.5, a response curve is developed for the submerged speed as a function of the factors test depth, payload length, PC, and shaft horsepower. Therefore, Equation (4) is reduced to the following

$$OMO E = fn(test\ depth, payload\ length, PC, shaft\ horsepower). \quad (5)$$

Using Equation (5), an OMOE response surface is created using the Central Composite design. The results are shown in Table 26 and 27, and Figure 22 and indicate an excellent model fit. As seen in Table 27, the payload length is the driver for the OMOE, followed by shaft horsepower and test depth.

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	14	1.2093301	0.086381	38817.67
Error	11	0.0000245	0.000002	Prob > F
C. Total	25	1.2093546		<.0001

Table 26: OMOE Response Model Analysis of Variance

### Actual by Predicted Plot

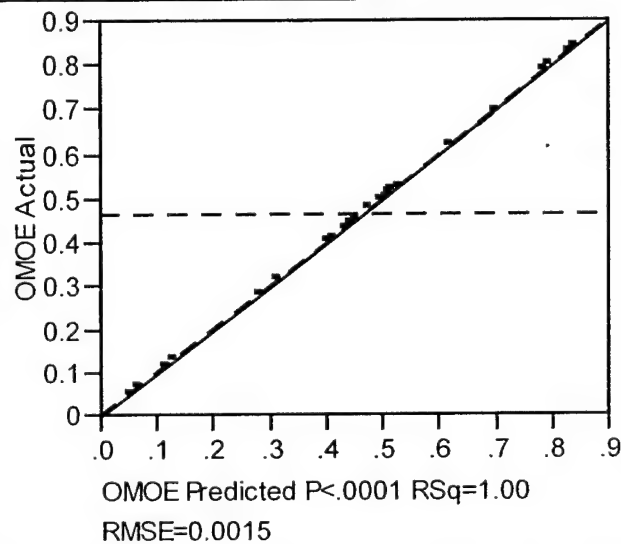


Figure 22: OMOE Response Model Leverage Plot

### Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob> t
Intercept	0.5045668	0.000591	853.25	<.0001
Depth(700,1100)&RS	0.0265937	0.000352	75.63	<.0001
Payload(65,88)&RS	0.1926272	0.000352	547.85	<.0001
SHP(20000,50000)&RS	0.1685787	0.000352	479.45	<.0001
Diameter(38,42)&RS	-0.007121	0.000352	-20.25	<.0001
Depth(700,1100)*Payload(65,88)	0.0003464	0.000373	0.93	0.3730
Depth(700,1100)*SHP(20000,50000)	-0.004939	0.000373	-13.24	<.0001
Payload(65,88)*SHP(20000,50000)	-0.000282	0.000373	-0.76	0.4651
Depth(700,1100)*Diameter(38,42)	0.0002758	0.000373	0.74	0.4751
Payload(65,88)*Diameter(38,42)	-0.000853	0.000373	-2.29	0.0430
SHP(20000,50000)*Diameter(38,42)	0.0021487	0.000373	5.76	0.0001
Depth(700,1100)*Depth(700,1100)	-0.001462	0.000932	-1.57	0.1450
Payload(65,88)*Payload(65,88)	0.0019499	0.000932	2.09	0.0605
SHP(20000,50000)*SHP(20000,50000)	-0.053365	0.000932	-57.25	<.0001
Diameter(38,42)*Diameter(38,42)	-0.000462	0.000932	-0.50	0.6301

Table 27: OMOE Response Model Coefficients



## 4.7 Design Space Trade-off Study

This chapter has generated a response surface for cost, submerged displacement, length, submerged speed, and OMOE. Using the contour profiler, the designer can now efficiently perform trade-off studies and examine the impact of the design factors on the submarine design.

In Figure 23, the designer has used the test depth factor as the x-axis and the shaft horsepower factor as the y-axis. The three other factors have been set as shown in Figure 23. If the designer has a cost constraint of \$2.0 billion and a minimum speed requirement of 28 knots, then the contour profiler can be used to set the corresponding constraints as shown in Figure 23. The net effect of the two constraints is a reduction in the feasible design space.

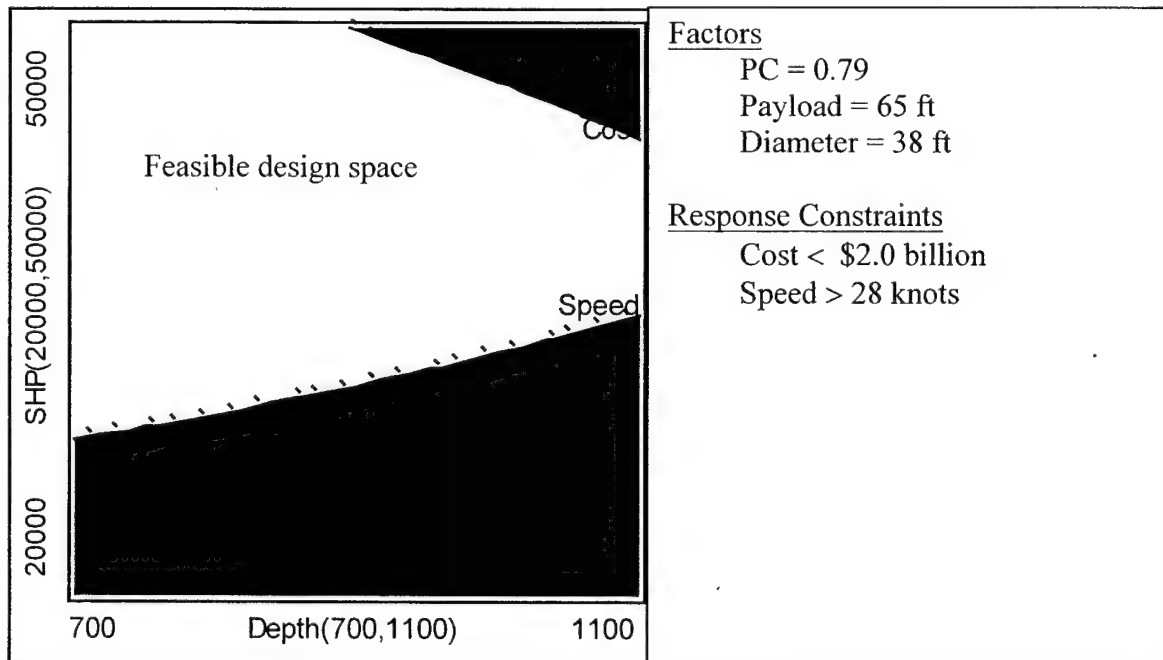


Figure 23: Trade-Off Analysis Part 1

To examine the impact of the diameter and payload length, both factors are increased to the maximum value as shown in Figure 24. The two combine to reduce the feasible design space.

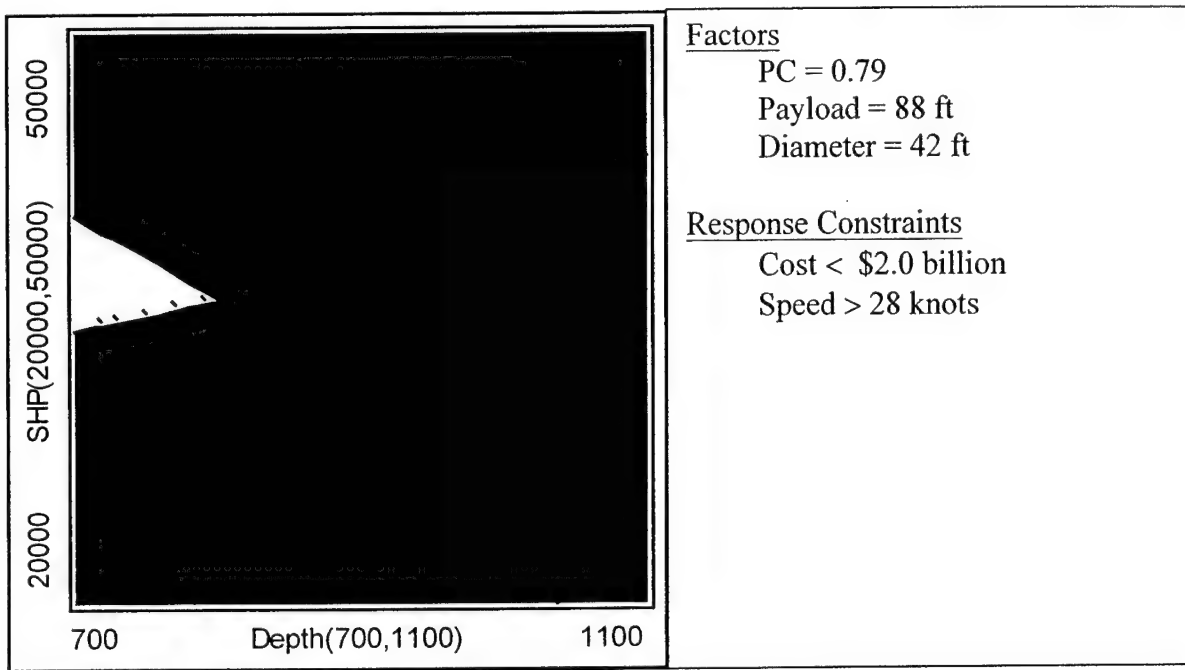


Figure 24: Trade-Off Analysis Part 2

Using the OMOE response, the designer can now determine the optimal OMOE at the cost constraint of \$2.0 billion. Using the contour profiler, Figure 25, the designer continues increasing the OMOE response contour until the contour just touches the \$2.0 billion cost contour. As shown in Figure 25, the optimal OMOE at this point is 0.72. Now the designer can change the other factors to determine if it is possible to increase the OMOE even further. Since the payload length is at the max value of 88 ft, the diameter is evaluated next.

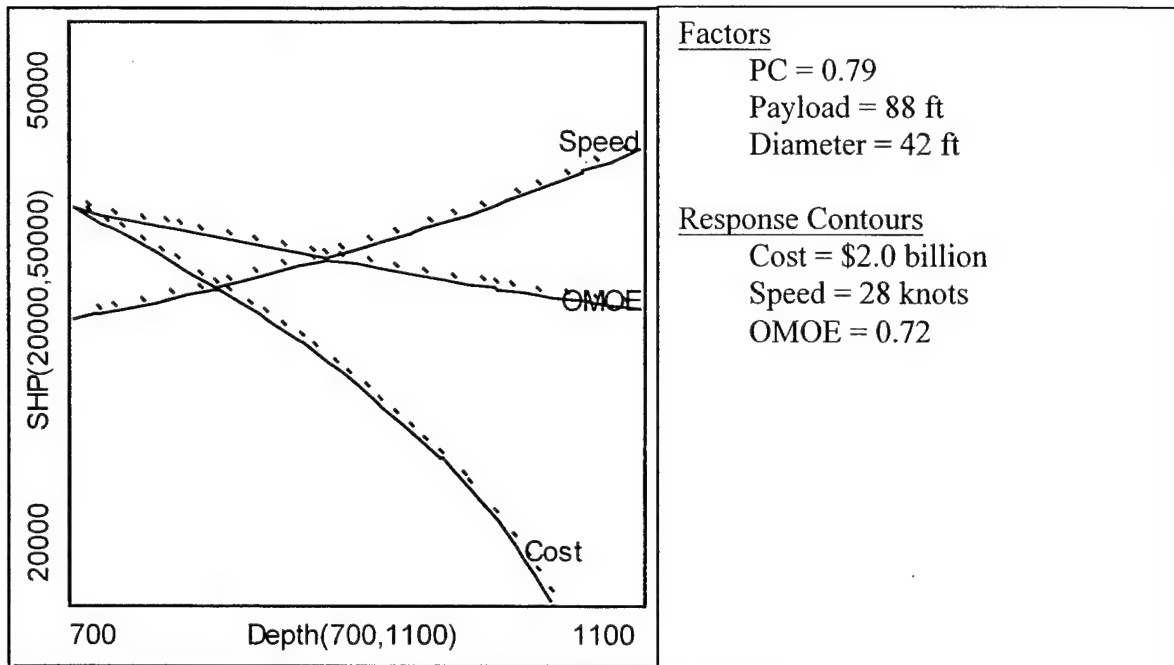


Figure 25: Trade-Off Analysis Part 3

In Figure 26, the diameter is reduced to 38 ft, resulting in an increase in the OMOE to a value of 0.784. This is the optimal OMOE with the \$2.0 billion cost constraint. In addition, as shown in Figure 27, reducing the diameter has increased the feasible design space. The designer has not only determined the optimal OMOE, but he has also mapped out the feasible design space.

With the overhead of creating the response surfaces, the designer has just determined the optimal concept design in a matter of seconds. And if the OMOE weightings change due to new customer preferences, the designer can update the JMP<sup>®</sup> file and, once again, determine the optimal OMOE in seconds. The ability to optimize the concept design is the benefit of combining RSM, a submarine model, and a statistical software package. The previous concept design methods outlined in Chapter One cannot efficiently perform the analysis conducted in this chapter.

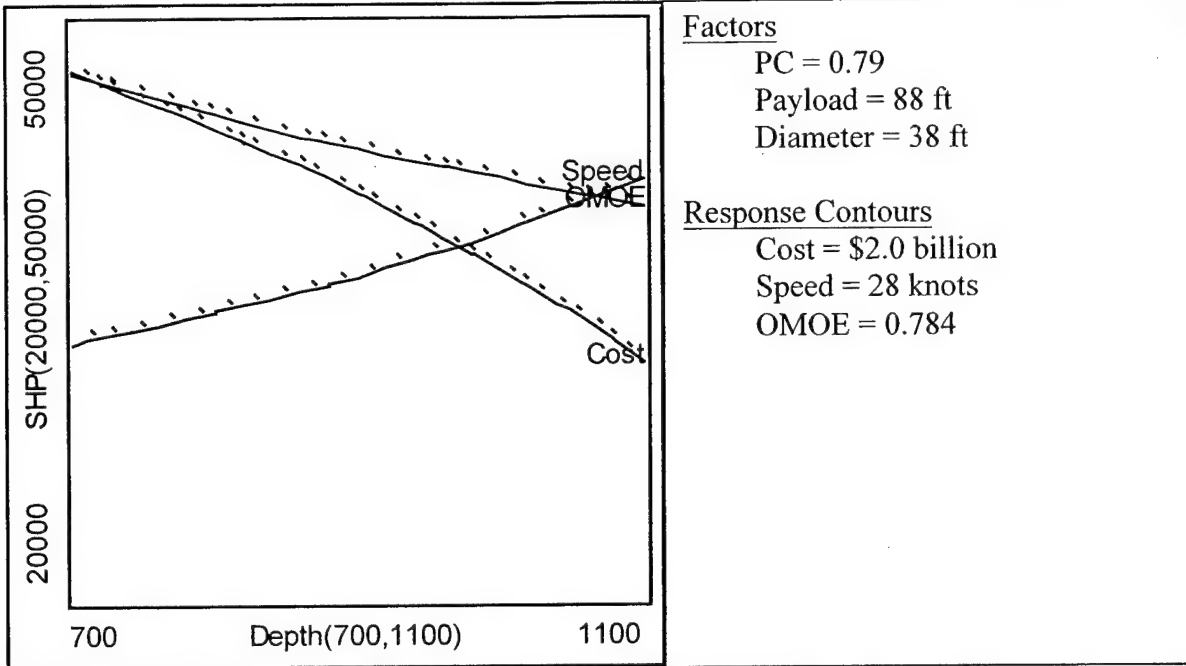


Figure 26: Trade-Off Analysis Part 4

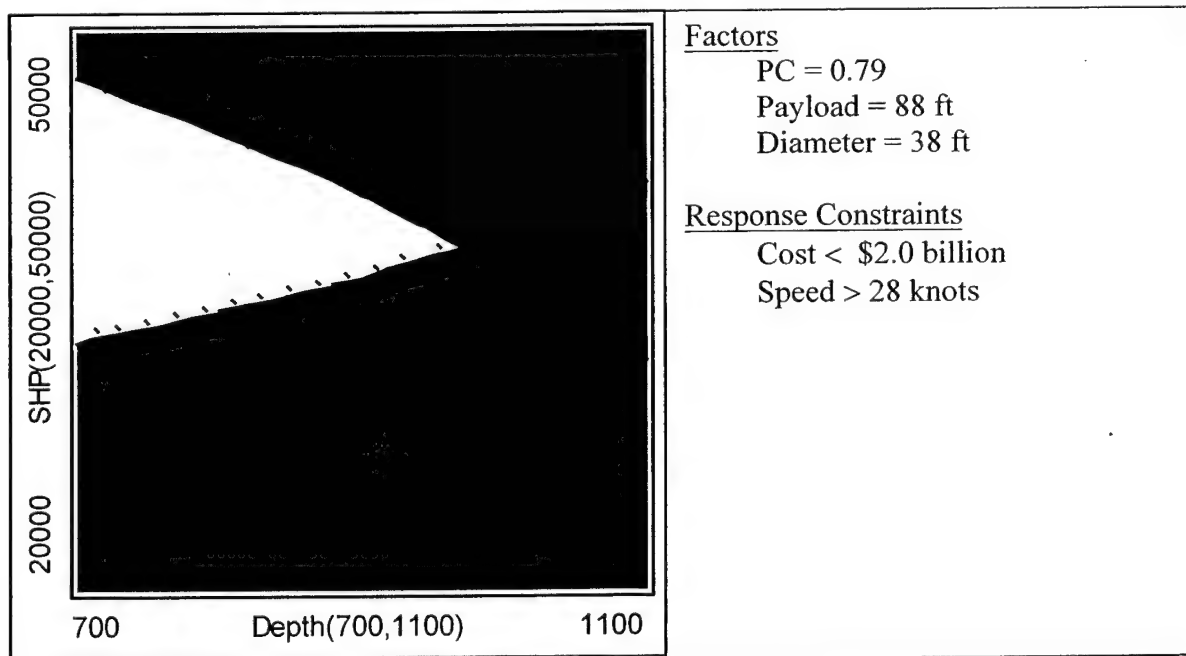


Figure 27: Trade-Off Analysis Part 5

Now if the designer is told to develop a concept design with the following characteristics

- Submerged speed  $> 28.0$  knots,
- Cost  $< \$2.0$  billion,
- Submerged Displacement  $< 10,000$  ltons,

then the designer enters in the new constraints into the program. As shown in Figure 28, the net result of the constraints is that there are no feasible concept designs that meet all the requirements. The only option is to reduce the payload length or increase PC. Now the fun begins!

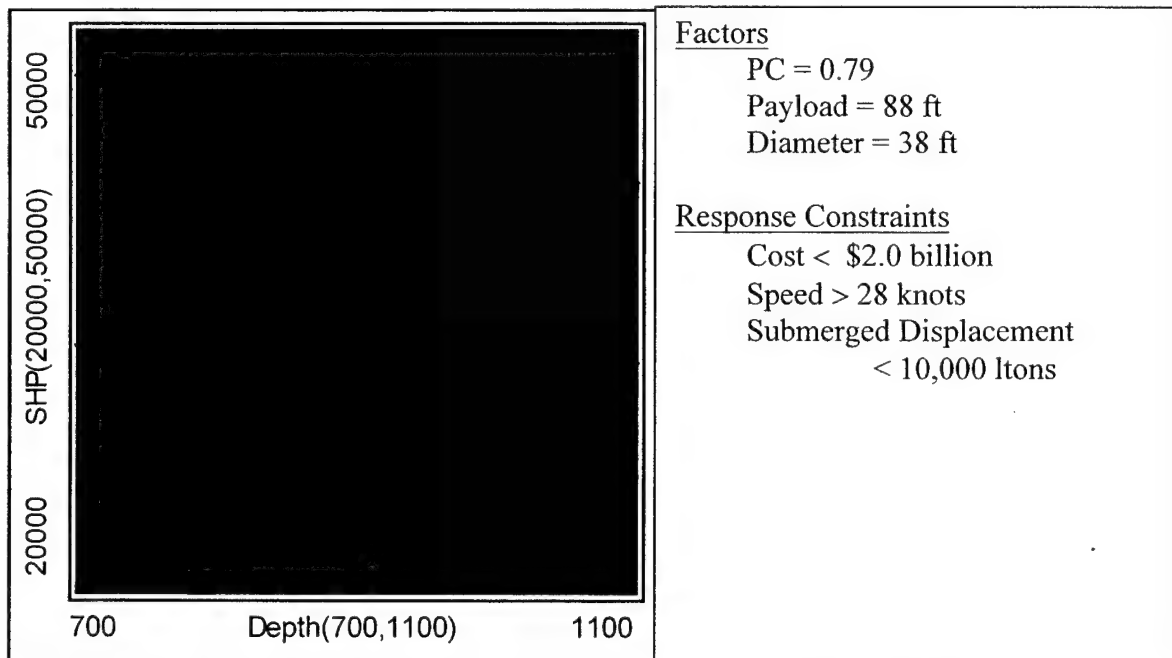


Figure 28: Trade-Off Analysis Part 6



## **Chapter 5: Conclusions**

Present submarine concept exploration is accomplished by either the one-factor-at-a-time design approach or the Pareto plot. Either method is guided primarily by past experience, rules-of-thumb, and designer preference. As described in Chapter One, this approach is ad hoc, not efficient, and may not lead to an optimum concept design. Furthermore, once the designer has a “good” concept design, he has no process or methodology to determine whether an improved concept design is possible or not. To improve upon current practices, a methodology is needed to formalize the design process and connect the customer preferences into the design process. As shown in this thesis, RSM can accomplish (and even surpass) this challenge.

### ***5.1 A New Concept Design Process***

To optimize the concept design, this thesis applies RSM to a basic submarine concept design. The methodology of RSM decomposes a complex design into a set of critical factors that can be used to model the responses of interest. The designer selects the responses he would like to model and determines the critical factors for each. Then

by applying RSM, the designer creates a response surface, which represents all feasible designs within the specified design space.

Using the methodology outlined in Chapter Two, a response surface was created for submerged speed, submerged displacement, length, cost, and OMOE in Chapter Four.

With the response surfaces created, the designer is able to accomplish the following:

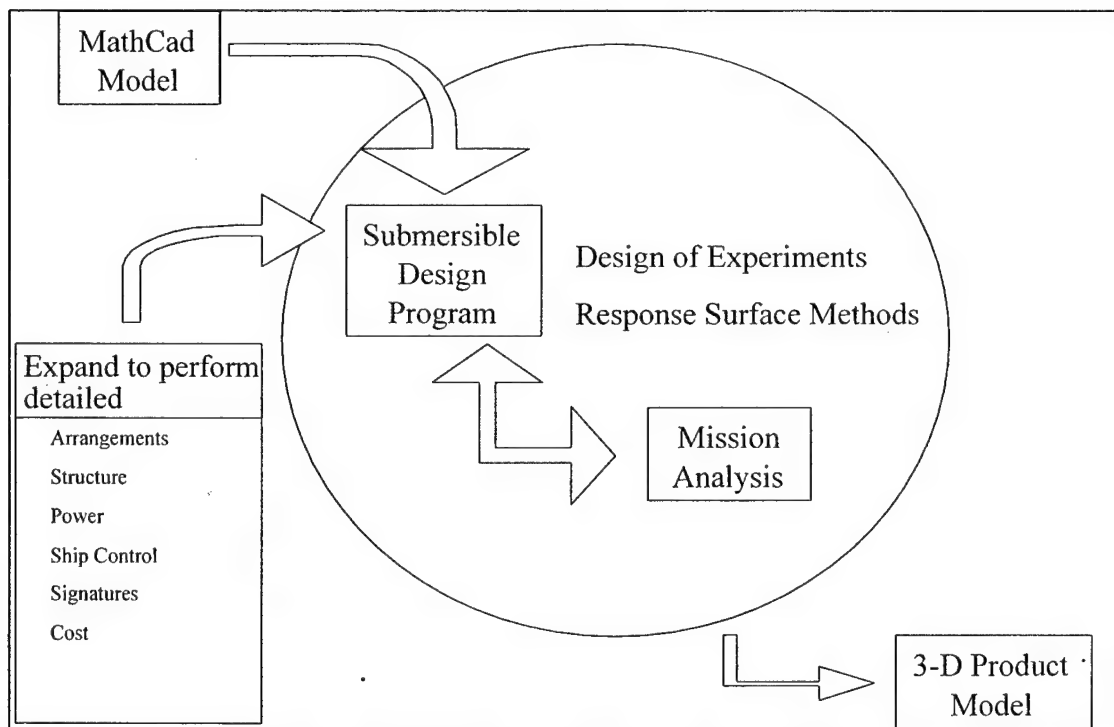
- Using the OMOE and cost response surface, determine the optimal OMOE for a specified cost.
- For each of the selected responses, the designer can efficiently examine an infinite number of combinations of the factors to determine the combined effect on the response. The response surface can aide the designer to determine which factors have the biggest and those that have the minimal impact on the response. With this information, the designer can focus his time and money on the key factors.
- By examining all the responses collectively, the designer can efficiently perform trade-off studies. This was demonstrated in Section 4.7 which examined the impact of the payload length on cost, submerged speed, submerged displacement, and OMOE.
- By setting upper and lower limits for the various responses, the designer can bracket his design space and determine the feasible design space. The designer can then instantly determine which combination of design factors are feasible and which are not. But more important, the designer can determine immediately if the imposed constraints result in a reduction of the design space to the point that there are no possible feasible designs.

## **5.2 Future Work**

This thesis outlines a methodology to optimize a concept design given a customer preference (via an OMOE). The Submersible Design Program is used to generate the design variants, which are then used to create the response surfaces, which are then used to optimize the design based on the OMOE. To further improve upon this work, future



research involves three general categories. First, expand the Submersible Design Program to provide more detailed concept design and capability. Second, incorporate a mission analysis tool to determine the mission effectiveness of the concept design. The mission analysis tool can be an integral part of the Submersible Design Program or a separate program/simulation model. Third, determine how to apply RSM to the combined Submersible Design Program and the mission analysis tool, as shown in Figure 29, resulting in an optimal solution based on customer preference and mission effectiveness. Specifically, the following additional work is needed:



**Figure 29: Future Submarine Concept Design Process**

### **5.2.1 Submersible Design Program**

The program provides a basic concept design, but more work is needed. Additional coding is required to provide more detailed calculations for the existing modules. Then additional modules need to be added to provide more information: structure analysis, detailed layout and space arrangements, ship controls and maneuverability, and acoustics/signatures.

### **5.2.2 Mission Analysis – OMOE**

To optimize the concept design, the designer needs guidance from the customer regarding what an optimal solution is. In linear and non-linear programming, this is in the form of an objective function. For submarine design, an OMOE is required that captures the customer requirements. But these requirements are extensive: submerged speed, test depth, cost, expendable payload, quieting, search rate, probability of kill, probability of counter kill, etc. Therefore, a detailed study should be completed that defines a generic OMOE (with the weighting specified by the designer) that can measure the performance and mission effectiveness of the submarine. And just as important, the characteristics of the OMOE must be an integral part of the Submersible Design Program.

### **5.2.3 Mission Analysis – Program/Simulation Model**

Another method to determine the effectiveness of the concept design is to use a program/simulation model already developed. The U.S. Navy has numerous programs and simulation analysis tools for submarine designs. For example, SIM II is used by the Naval Undersea Warfare Center, Newport, Rhode Island, to investigate submarine

sensors, weapons, platform characteristics, communications, countermeasures, and tactics. A study should be completed to determine if it is possible to connect the Submersible Design Program output to one of the mission analysis simulation/programs.

#### **5.2.4 Design of Experiments/Response Surface Methods**

Another research area is the application of RSM to the overall concept design process. This thesis applies RSM to the modular submarine model. As shown in Figure 29, RSM could be applied to the various mission analysis tools. Therefore, research is needed to determine how to apply RSM to the combined Submersible Design Program and the mission analysis tool, resulting in an optimal solution based on customer preference and mission effectiveness.



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# Appendix A

## Hanson and Hunt

### Box-Behnken Design Matrix

Pattern	Diameter (ft)	Depth (ft)	Speed (knots)	Payload Length (ft)	SHP	Cost (\$million)	GMt (ft)	Extra Volume (ft <sup>3</sup> )
0000	40	800	28	65.5	28,931	1,540.3	1.10	37,500.0
+00-	42	800	28	43	24,648	1,287.1	1.05	8,135.9
-00+	38	800	28	88	32,672	1,745.9	1.12	61,828.6
0+-0	40	1100	24	65.5	19,936	1,644.8	1.10	77,949.6
+0-0	42	800	24	65.5	18,038	1,476.8	1.08	33,725.4
+--00	42	500	28	65.5	26,541	1,406.1	1.10	382.5
--00	38	500	28	65.5	25,873	1,298.3	1.13	-6,113.8
0++0	40	1100	32	65.5	49,230	2,195.7	1.12	108,333.9
00++	40	800	32	88	55,132	2,445.0	1.10	81,272.6
+0+0	42	800	32	65.5	51,685	2,139.7	1.06	71,659.6
+00+	42	800	28	88	34,760	1,965.4	1.12	83,976.0
-0-0	38	800	24	65.5	16,919	1,317.0	1.10	18,076.5
0-0-	40	500	28	43	23,853	1,157.9	1.04	-16,982.9
0000	40	800	28	65.5	28,931	1,540.3	1.10	37,500.0
0+0+	40	1100	28	88	39,108	2,230.8	1.10	146,695.5
-0+0	38	800	32	65.5	47,905	1,851.5	1.07	52,888.8
-00-	38	800	28	43	23,697	1,187.8	1.09	-1,392.1
0-0+	40	500	28	88	29,822	1,593.6	1.12	19,173.2
-+00	38	1100	28	65.5	32,318	1,728.8	1.08	84,775.9
0-+0	40	500	32	65.5	41,323	1,549.5	1.06	8,556.9
00-+	40	800	24	88	20,424	1,689.8	1.13	58,071.4
00--	40	800	24	43	14,658	1,109.7	1.11	-3,447.3
00+-	40	800	32	43	38,756	1,431.3	1.07	19,494.6
0--0	40	500	24	65.5	16,203	1,242.2	1.11	-7,476.0
++00	42	1100	28	65.5	34,113	1,925.1	1.09	108,475.2
0000	40	800	28	65.5	28,931	1,540.3	1.10	37,500.0
0+0-	40	1100	28	43	27,371	1,444.4	1.11	46,883.8









# Appendix C

## Cost, Submerged Displacement and LOA Response Surface -

### Central Composite Design Matrix

Pattern	Depth (ft)	Payload Length (ft)	SHP	Diameter (ft)	Cost (\$million)	Submerged Displacement (ltons)	LOA (ft)
++++	1100	88	50,000	42	2691.4	17,064	553.3
---+	700	88	50,000	38	2075.2	12,421	494.8
+--+	1100	65	20,000	42	1725.7	11,136	392.2
--++	700	65	50,000	42	1987.8	11,739	408.3
00a0	900	76.5	20,000	40	1627.3	10,376	397.9
000a	900	76.5	35,000	38	1735.4	10,784	439.9
++--	1100	88	20,000	38	1836.7	12,001	481.2
A000	1100	76.5	35,000	40	2000.9	12,785	468.6
---+	700	88	20,000	38	1524.0	9,592	403.0
00A0	900	76.5	50,000	40	2213.1	13,472	488.7
+++--	1100	88	50,000	38	2464.0	15,412	592.0
0a00	900	65	35,000	40	1684.5	10,387	396.5
+---	1100	65	20,000	38	1557.5	9,912	409.9
+--+	1100	65	50,000	42	2353.1	14,549	483.0
---+	700	65	20,000	42	1436.8	8,912	333.1
0000	900	76.5	35,000	40	1820.7	11,398	427.9
+--+	1100	65	50,000	38	2184.8	13,325	520.8
--+-	700	65	50,000	38	1853.9	10,779	438.1
-+++	700	88	50,000	42	2255.7	13,710	464.1
000A	900	76.5	35,000	42	1910.6	12,045	418.1
a000	700	76.5	35,000	40	1672.3	10,256	394.4
++-+	1100	88	20,000	42	2063.8	13,652	462.6
0000	900	76.5	35,000	40	1820.7	11,398	427.9
----	700	65	20,000	38	1303.1	7,954	346.3
-+-+	700	88	20,000	42	1704.2	10,879	388.9
0A00	900	88	35,000	40	1957.1	12,409	459.3



# Appendix D

## Submerged Speed Screening Experiment -

### Fractional Factorial (Resolution IV) Design Matrix

Pattern	D (ft)	TD (ft)	SHP	PL (ft)	Entrance	Run	Sail Area (ft <sup>2</sup> )	Appendage Area (ft)	PC	Submerged Speed (knots)
+++--+--	42	1100	50,000	65	3.5	1.75	285	3,500	0.79	30.05
++-++-+-	42	1100	45,000	85	3.5	1.75	575	2,000	0.79	28.84
-+--++++	38	1100	45,000	65	3.5	4	575	3,500	0.85	30.36
-+-+---+	38	1100	45,000	85	3.5	1.75	285	3,500	0.79	28.63
---++-+-	38	700	45,000	85	3.5	1.75	575	2,000	0.85	31.50
+++--++-	42	1100	45,000	65	3.5	4	285	2,000	0.85	31.11
-++-+-+-	38	1100	50,000	65	1.75	4	285	3,500	0.79	30.20
-++-+-+-	38	1100	50,000	65	3.5	1.75	575	2,000	0.79	30.65
--+++++-	38	700	50,000	85	3.5	4	575	3,500	0.79	30.70
--+-+++-	38	700	50,000	65	3.5	1.75	285	3,500	0.85	32.52
+++--++-	42	1100	50,000	65	1.75	4	575	2,000	0.79	30.61
+--+--++	42	700	50,000	65	1.75	4	285	3,500	0.85	32.47
+-----	42	700	50,000	85	3.5	4	285	2,000	0.79	31.41
+--+--++	42	700	50,000	85	1.75	1.75	575	3,500	0.79	30.34
--+-+++-	38	700	50,000	65	1.75	4	575	2,000	0.85	33.21
-+-+---+	38	1100	45,000	85	1.75	4	575	2,000	0.79	29.12
+++++++-	42	1100	50,000	85	3.5	4	575	3,500	0.85	29.73
++-+-+--	42	1100	45,000	85	1.75	4	285	3,500	0.79	28.45
+-----	42	700	45,000	65	1.75	1.75	285	2,000	0.79	31.71
+---+++-	42	700	45,000	65	3.5	4	575	3,500	0.79	30.98
-----+	38	700	45,000	65	1.75	1.75	575	3,500	0.79	30.90
++-----	42	1100	45,000	65	1.75	1.75	575	3,500	0.85	30.02
+---+++-	42	700	45,000	85	1.75	4	575	2,000	0.85	31.30
---+-+--	38	700	45,000	85	1.75	4	285	3,500	0.85	30.97
-++++--+	38	1100	50,000	85	3.5	4	285	2,000	0.85	30.62
+---+++-	42	700	45,000	85	3.5	1.75	285	3,500	0.85	30.68
+--+--++	42	700	50,000	65	3.5	1.75	575	2,000	0.85	33.02
-++++--+	38	1100	50,000	85	1.75	1.75	575	3,500	0.85	29.75
-+-----	38	1100	45,000	65	1.75	1.75	285	2,000	0.85	31.03
++++-----	42	1100	50,000	85	1.75	1.75	285	2,000	0.85	30.25



# Appendix E

## Submerged Speed Response Surface - Central Composite Design Matrix

Pattern	Test Depth (ft)	Payload Length (ft)	PC	SHP	Submerged Speed (knots)
---+	700	65	0.79	50,000	31.91
++--	1100	65	0.79	50,000	30.10
++--	1100	65	0.85	50,000	30.87
--++	700	65	0.85	50,000	32.73
0000	900	76.5	0.82	35,000	28.45
++--	1100	88	0.79	20,000	22.41
++--	1100	88	0.79	50,000	28.78
----	700	88	0.85	50,000	31.35
----	700	88	0.79	50,000	30.57
00a0	900	76.5	0.79	35,000	28.08
000A	900	76.5	0.82	50,000	30.73
000a	900	76.5	0.82	20,000	24.06
-+--	700	88	0.79	20,000	23.81
+--+	1100	65	0.85	20,000	24.31
a000	700	76.5	0.82	35,000	29.27
A000	1100	76.5	0.82	35,000	27.57
----	700	65	0.79	20,000	25.14
0000	900	76.5	0.82	35,000	28.45
0a00	900	65	0.82	35,000	29.21
--+-	700	65	0.85	20,000	25.78
++++	1100	88	0.85	50,000	29.52
0A00	900	88	0.82	35,000	27.76
00A0	900	76.5	0.85	35,000	28.80
+++--	1100	88	0.85	20,000	22.98
+---	1100	65	0.79	20,000	23.71